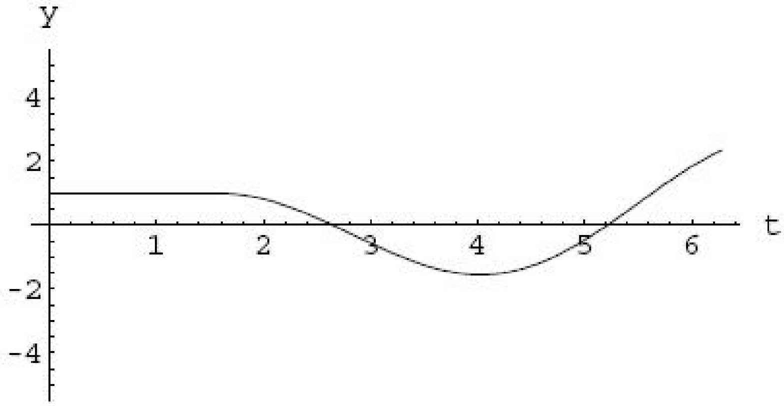
SEC 4.4

5. $\mathscr{L}\{t^2 \sinh t\} = \dfrac{d^2}{ds^2}\left(\dfrac{1}{s^2-1}\right) = \dfrac{6s^2+2}{(s^2-1)^3}$

16.



19. $\mathscr{L}\{1 * t^3\} = \dfrac{1}{s}\,\dfrac{3!}{s^4} = \dfrac{6}{s^5}$

23. $\mathscr{L}\left\{\displaystyle\int_0^t e^{\tau}\,d\tau\right\} = \dfrac{1}{s}\,\mathscr{L}\{e^t\} = \dfrac{1}{s(s-1)}$

34. Using $\mathscr{L}^{-1}\left\{\dfrac{1}{(s-a)^2}\right\} = te^{at}$, (8) in the text gives

$$\mathscr{L}^{-1}\left\{\frac{1}{s(s-a)^2}\right\} = \int_0^t \tau e^{a\tau}\,d\tau = \frac{1}{a^2}(ate^{at} - e^{at} + 1).$$

45. The Laplace transform of the given equation is

$$s\,\mathscr{L}\{y\} - y(0) = \mathscr{L}\{1\} - \mathscr{L}\{\sin t\} - \mathscr{L}\{1\}\,\mathscr{L}\{y\}.$$

Solving for $\mathscr{L}\{f\}$ we obtain

$$\mathscr{L}\{y\} = \frac{s^2-s+1}{(s^2+1)^2} = \frac{1}{s^2+1} - \frac{1}{2}\,\frac{2s}{(s^2+1)^2}\,.$$

Thus

$$y = \sin t - \frac{1}{2}\,t\sin t.$$