SEC 4.3

8. $\mathscr{L}\{e^{-2t}\cos 4t\} = \dfrac{s+2}{(s+2)^2+16}$

13. $\mathscr{L}^{-1}\left\{\dfrac{1}{s^2-6s+10}\right\} = \mathscr{L}^{-1}\left\{\dfrac{1}{(s-3)^2+1^2}\right\} = e^{3t}\sin t$

20. $\mathscr{L}^{-1}\left\{\dfrac{(s+1)^2}{(s+2)^4}\right\} = \mathscr{L}^{-1}\left\{\dfrac{1}{(s+2)^2} - \dfrac{2}{(s+2)^3} + \dfrac{1}{6}\dfrac{3!}{(s+2)^4}\right\} = te^{-2t} - t^2e^{-2t} + \dfrac{1}{6}t^3e^{-2t}$

24. The Laplace transform of the differential equation is

$$s^2\,\mathscr{L}\{y\} - sy(0) - y'(0) - 4\left[s\,\mathscr{L}\{y\} - y(0)\right] + 4\,\mathscr{L}\{y\} = \frac{6}{(s-2)^4}.$$

Solving for $\mathscr{L}\{y\}$ we obtain $\mathscr{L}\{y\} = \dfrac{1}{20}\dfrac{5!}{(s-2)^6}$. Thus, $y = \dfrac{1}{20}t^5e^{2t}$.

47. $\mathscr{L}^{-1}\left\{\dfrac{e^{-s}}{s(s+1)}\right\} = \mathscr{L}^{-1}\left\{\dfrac{e^{-s}}{s} - \dfrac{e^{-s}}{s+1}\right\} = \mathscr{U}(t-1) - e^{-(t-1)}\,\mathscr{U}(t-1)$

60. $\mathscr{L}\{\sin t - \sin t\;\mathscr{U}(t-2\pi)\} = \mathscr{L}\{\sin t - \sin(t-2\pi)\,\mathscr{U}(t-2\pi)\} = \dfrac{1}{s^2+1} - \dfrac{e^{-2\pi s}}{s^2+1}$

66. The Laplace transform of the differential equation is

$$s^2\,\mathscr{L}\{y\} - sy(0) - y'(0) + 4\,\mathscr{L}\{y\} = \frac{1}{s} - \frac{e^{-s}}{s}.$$

Solving for $\mathscr{L}\{y\}$ we obtain

$$\mathscr{L}\{y\} = \frac{1-s}{s(s^2+4)} - e^{-s}\frac{1}{s(s^2+4)} = \frac{1}{4}\frac{1}{s} - \frac{1}{4}\frac{s}{s^2+4} - \frac{1}{2}\frac{2}{s^2+4} - e^{-s}\left[\frac{1}{4}\frac{1}{s} - \frac{1}{4}\frac{s}{s^2+4}\right].$$

Thus

$$y = \frac{1}{4} - \frac{1}{4}\cos 2t - \frac{1}{2}\sin 2t - \left[\frac{1}{4} - \frac{1}{4}\cos 2(t-1)\right]\mathscr{U}(t-1).$$