SEC 4.2

5. $\mathscr{L}^{-1}\left\{\frac{(s+1)^3}{s^4}\right\} = \mathscr{L}^{-1}\left\{\frac{1}{s} + 3\cdot\frac{1}{s^2} + \frac{3}{2}\cdot\frac{2}{s^3} + \frac{1}{6}\cdot\frac{3!}{s^4}\right\} = 1 + 3t + \frac{3}{2}t^2 + \frac{1}{6}t^3$

13. $\mathscr{L}^{-1}\left\{\frac{4s}{4s^2+1}\right\} = \mathscr{L}^{-1}\left\{\frac{s}{s^2+1/4}\right\} = \cos\frac{1}{2}t$

15. $\mathscr{L}^{-1}\left\{\frac{2s-6}{s^2+9}\right\} = \mathscr{L}^{-1}\left\{2\cdot\frac{s}{s^2+9} - 2\cdot\frac{3}{s^2+9}\right\} = 2\cos 3t - 2\sin 3t$

19. $\mathscr{L}^{-1}\left\{\frac{s}{s^2+2s-3}\right\} = \mathscr{L}^{-1}\left\{\frac{1}{4}\cdot\frac{1}{s-1} + \frac{3}{4}\cdot\frac{1}{s+3}\right\} = \frac{1}{4}e^t + \frac{3}{4}e^{-3t}$

35. The Laplace transform of the initial-value problem is

$$s^2\mathscr{L}\{y\} - sy(0) - y'(0) + 5\,[s\mathscr{L}\{y\} - y(0)] + 4\,\mathscr{L}\{y\} = 0.$$

Solving for $\mathscr{L}\{y\}$ we obtain

$$\mathscr{L}\{y\} = \frac{s+5}{s^2+5s+4} = \frac{4}{3}\,\frac{1}{s+1} - \frac{1}{3}\,\frac{1}{s+4}\,.$$

Thus

$$y = \frac{4}{3}e^{-t} - \frac{1}{3}e^{-4t}.$$

36. The Laplace transform of the initial-value problem is

$$s^2\mathscr{L}\{y\} - sy(0) - y'(0) - 4\,[s\mathscr{L}\{y\} - y(0)] = \frac{6}{s-3} - \frac{3}{s+1}\,.$$

Solving for $\mathscr{L}\{y\}$ we obtain

$$\begin{aligned}\mathscr{L}\{y\} &= \frac{6}{(s-3)(s^2-4s)} - \frac{3}{(s+1)(s^2-4s)} + \frac{s-5}{s^2-4s}\\ &= \frac{5}{2}\cdot\frac{1}{s} - \frac{2}{s-3} - \frac{3}{5}\cdot\frac{1}{s+1} + \frac{11}{10}\cdot\frac{1}{s-4}\,.\end{aligned}$$

Thus

$$y = \frac{5}{2} - 2e^{3t} - \frac{3}{5}e^{-t} + \frac{11}{10}e^{4t}\,.$$