SEC 4.1

3. $$\mathscr{L}\{f(t)\} = \int_0^1 te^{-st}dt + \int_1^\infty e^{-st}dt = \left(-\frac{1}{s}te^{-st} - \frac{1}{s^2}e^{-st}\right)\Big|_0^1 - \frac{1}{s}e^{-st}\Big|_1^\infty$$
$$= \left(-\frac{1}{s}e^{-s} - \frac{1}{s^2}e^{-s}\right) - \left(0 - \frac{1}{s^2}\right) - \frac{1}{s}(0 - e^{-s}) = \frac{1}{s^2}(1 - e^{-s}), \quad s > 0$$

5. $$\mathscr{L}\{f(t)\} = \int_0^\pi (\sin t)e^{-st}dt = \left(-\frac{s}{s^2+1}e^{-st}\sin t - \frac{1}{s^2+1}e^{-st}\cos t\right)\Big|_0^\pi$$
$$= \left(0 + \frac{1}{s^2+1}e^{-\pi s}\right) - \left(0 - \frac{1}{s^2+1}\right) = \frac{1}{s^2+1}(e^{-\pi s} + 1), \quad s > 0$$

17. $$\mathscr{L}\{f(t)\} = \int_0^\infty t(\cos t)e^{-st}dt$$
$$= \left[\left(-\frac{st}{s^2+1} - \frac{s^2-1}{(s^2+1)^2}\right)(\cos t)e^{-st} + \left(\frac{t}{s^2+1} + \frac{2s}{(s^2+1)^2}\right)(\sin t)e^{-st}\right]_0^\infty$$
$$= \frac{s^2-1}{(s^2+1)^2}, \quad s > 0$$

29. $$\mathscr{L}\{1 + 2e^{2t} + e^{4t}\} = \frac{1}{s} + \frac{2}{s-2} + \frac{1}{s-4}$$

38. $$\mathscr{L}\{\cos^2 t\} = \mathscr{L}\left\{\frac{1}{2} + \frac{1}{2}\cos 2t\right\} = \frac{1}{2s} + \frac{1}{2}\,\frac{s}{s^2+4}$$

40. From the addition formula for the cosine function,

$$\cos\left(t - \frac{\pi}{6}\right) = \cos t \cos\frac{\pi}{6} + \sin t \sin\frac{\pi}{6} = \frac{\sqrt{3}}{2}\cos t + \frac{1}{2}\sin t$$

so

$$\mathscr{L}\left\{\cos\left(t - \frac{\pi}{6}\right)\right\} = \frac{\sqrt{3}}{2}\mathscr{L}\{\cos t\} + \frac{1}{2}\mathscr{L}\{\sin t\}$$
$$= \frac{\sqrt{3}}{2}\,\frac{s}{s^2+1} + \frac{1}{2}\,\frac{1}{s^2+1} = \frac{1}{2}\,\frac{\sqrt{3}\,s+1}{s^2+1}.$$