Sec 15.3

4. From formulas (5) and (6) in the text,

$$
\begin{aligned}
A(\alpha) &= \int_{-\infty}^{\infty} f(x)\cos\alpha x\,dx \\
&= \int_{-\infty}^{0} 0\cdot\cos\alpha x\,dx + \int_{0}^{\pi} \sin x\cos\alpha x\,dx + \int_{\pi}^{\infty} 0\cdot\cos\alpha x\,dx \\
&= \frac{1}{2}\int_{0}^{\pi} [\sin(1+\alpha)x + \sin(1-\alpha)x]\,dx \\
&= \frac{1}{2}\left[-\frac{\cos(1+\alpha)x}{1+\alpha} - \frac{\cos(1-\alpha)x}{1-\alpha}\right]_0^{\pi} \\
&= -\frac{1}{2}\left[\frac{\cos(1+\alpha)\pi - 1}{1+\alpha} + \frac{\cos(1-\alpha)\pi - 1}{1-\alpha}\right] \\
&= -\frac{1}{2}\left[\frac{\cos(1+\alpha)\pi - \alpha\cos(1+\alpha)\pi + \cos(1-\alpha)\pi + \alpha\cos(1-\alpha)\pi - 2}{1-\alpha^2}\right] \\
&= \frac{1+\cos\alpha\pi}{1-\alpha^2},
\end{aligned}
$$

and

$$
\begin{aligned}
B(\alpha) &= \int_0^{\pi} \sin x\sin\alpha x\,dx = \frac{1}{2}\int_0^{\pi} [\cos(1-\alpha)x - \cos(1+\alpha)]\,dx \\
&= \frac{1}{2}\left[\frac{\sin(1-\alpha)\pi}{1-\alpha} - \frac{\sin(1+\alpha)\pi}{1+\alpha}\right] = \frac{\sin\alpha\pi}{1-\alpha^2}.
\end{aligned}
$$

Hence

$$
\begin{aligned}
f(x) &= \frac{1}{\pi}\int_0^{\infty} \frac{\cos\alpha x + \cos\alpha x\cos\alpha\pi + \sin\alpha x\sin\alpha\pi}{1-\alpha^2}\,d\alpha \\
&= \frac{1}{\pi}\int_0^{\infty} \frac{\cos\alpha x + \cos\alpha(x-\pi)}{1-\alpha^2}\,d\alpha.
\end{aligned}
$$

8. The function is even. Thus from formula (9) in the text

$$A(\alpha) = \pi \int_1^2 \cos \alpha x \, dx = \pi \left(\frac{\sin 2\alpha - \sin \alpha}{\alpha} \right)$$

Hence from formula (8) in the text,

$$f(x) = 2 \int_0^\infty \frac{(\sin 2\alpha - \sin \alpha) \cos \alpha x}{\alpha} \, d\alpha.$$

12. The function is odd. Thus from formula (11) in the text

$$B(\alpha) = \int_0^\infty x e^{-x} \sin \alpha x \, dx.$$

Now recall

$$\mathscr{L}\{t \sin kt\} = -\frac{d}{ds} \mathscr{L}\{\sin kt\} = 2ks/(s^2 + k^2)^2.$$

If we set $s = 1$ and $k = \alpha$ we obtain

$$B(\alpha) = \frac{2\alpha}{(1 + \alpha^2)^2} \, .$$

Hence from formula (10) in the text

$$f(x) = \frac{4}{\pi} \int_0^\infty \frac{\alpha \sin \alpha x}{(1 + \alpha^2)^2} \, d\alpha.$$

18. From the formula for sine integral of $f(x)$ we have

$$\begin{aligned} f(x) &= \frac{2}{\pi} \int_0^\infty \left(\int_0^\infty f(x) \sin \alpha x \, dx \right) \sin \alpha x \, dx \\ &= \frac{2}{\pi} \left[\int_0^1 1 \cdot \sin \alpha x \, d\alpha + \int_1^\infty 0 \cdot \sin \alpha x \, d\alpha \right] \\ &= \frac{2}{\pi} \, \frac{(-\cos \alpha x)}{x} \Bigg|_0^1 = \frac{2}{\pi} \, \frac{1 - \cos x}{x} \, . \end{aligned}$$