Sec 12.3

4. Since $f(-x) = (-x)^3 + 4x = -(x^3 - 4x) = -f(x)$, $f(x)$ is an odd function.

5. Since $f(-x) = e^{|-x|} = e^{|x|} = f(x)$, $f(x)$ is an even function.

6. Since $f(-x) = e^{-x} - e^{x} = -f(x)$, $f(x)$ is an odd function.

14. Since $f(x)$ is an odd function, we expand in a sine series:

$$b_n = \frac{2}{\pi}\int_0^{\pi} x \sin nx\, dx = \frac{2}{n}(-1)^{n+1}.$$

Thus

$$f(x) = \sum_{n=1}^{\infty} \frac{2}{n}(-1)^{n+1} \sin nx.$$

16. Since $f(x)$ is an odd function, we expand in a sine series:

$$b_n = 2\int_0^1 x^2 \sin n\pi x\, dx = 2\left(-\frac{x^2}{n\pi}\cos n\pi x\,\Big|_0^1 + \frac{2}{n\pi}\int_0^1 x\cos n\pi x\, dx\right) = \frac{2(-1)^{n+1}}{n\pi} + \frac{4}{n^3\pi^3}[(-1)^n - 1].$$

Thus

$$f(x) = \sum_{n=1}^{\infty}\left(\frac{2(-1)^{n+1}}{n\pi} + \frac{4}{n^3\pi^3}[(-1)^n - 1]\right)\sin n\pi x.$$

26. $a_0 = 2\int_{1/2}^{1} 1\, dx = 1$

$$a_n = 2\int_{1/2}^{1} 1\cdot\cos n\pi x\, dx = -\frac{2}{n\pi}\sin\frac{n\pi}{2}$$

$$b_n = 2\int_{1/2}^{1} 1\cdot\sin n\pi x\, dx = \frac{2}{n\pi}\left(\cos\frac{n\pi}{2} + (-1)^{n+1}\right)$$

$$f(x) = \frac{1}{2} + \sum_{n=1}^{\infty}\left(-\frac{2}{n\pi}\sin\frac{n\pi}{2}\right)\cos n\pi x$$

$$f(x) = \sum_{n=1}^{\infty}\frac{2}{n\pi}\left(\cos\frac{n\pi}{2} + (-1)^{n+1}\right)\sin n\pi x$$

38. $a_0 = \frac{2}{2}\int_0^2 (2-x)\,dx = 2$

$$a_n = \frac{2}{2}\int_0^2 (2-x)\cos n\pi x\,dx = 0$$

$$b_n = \frac{2}{2}\int_0^2 (2-x)\sin n\pi x\,dx = \frac{2}{n\pi}$$

$$f(x) = 1 + \sum_{n=1}^{\infty} \frac{2}{n\pi}\sin n\pi x$$