Sec 9.9

7. (a) $P_y = 4xy = Q_x$ and the integral is independent of path. $\phi_x = 2y^2x - 3$, $\phi = x^2y^2 - 3x + g(y)$, $\phi_y = 2x^2y + g'(y) = 2x^2y + 4$, $g(y) = 4y$, $\phi = x^2y^2 - 3x + 4y$,

$$\int_{(1,2)}^{(3,6)} (2y^2x - 3)\,dx + (2yx^2 + 4)\,dy = (x^2y^2 - 3x + 4y)\Big|_{(1,2)}^{(3,6)} = 330$$

(b) Use $y = 2x$ for $1 \le x \le 3$.

$$\int_{(1,2)}^{(3,6)} (2y^2x - 3)\,dx + (2yx^2 + 4)\,dy = \int_1^3 \left([2(2x)^2x - 3] + [2(2x)x^2 + 4]2\right)\,dx$$

$$= \int_1^3 (16x^3 + 5)\,dx = (4x^4 + 5x)\Big|_1^3 = 330$$

15. $P_y = 1 = Q_x$ and the vector field is a gradient field. $\phi_x = x^3 + y$, $\phi = \frac{1}{4}x^4 + xy + g(y)$, $\phi_y = x + g'(y) = x + y^3$, $g(y) - \frac{1}{4}y^4$, $\phi = \frac{1}{4}x^4 + xy + \frac{1}{4}y^4$

18. Since $P_y = -e^{-y} = Q_x$, $\mathbf{F}$ is conservative and $\int_C \mathbf{F}\cdot d\mathbf{r}$ is independent of the path. Thus, instead of the given curve we may use the simpler curve C_1: $y = 0$, $-2 \le -x \le 2$. Then $dy = 0$ and

$$W = \int_{C_1} (2x + e^{-y})\,dx + (4y - xe^{-y})\,dy = \int_2^{-2} (2x + 1)\,dx = (x^2 + x)\Big|_2^{-2} = (4 - 2) - (4 + 2) = -4.$$

23. $P_y = 0 = Q_x$, $Q_z = 0 = R_y$, $R_x = 2e^{2z} = P_z$ and the integral is independent of path. Parameterize the line segment between the points by $x = 1 + t$, $y = 1 + t$, $z = \ln 3$, for $0 \le t \le 1$. Then $dx = dy = dt$, $dz = 0$ and

$$\int_{(1,1,\ln 3)}^{(2,2\ln 3)} e^{2z}\,dx + 3y^2\,dy + 2xe^{2z}\,dz = \int_0^1 [e^{2\ln 3} + 3(1+t)^2]\,dt = [9t + (1+t)^3]\Big|_0^1 = 16.$$

28. Since $P_y = 24xy^2z = Q_x$, $Q_z = 12x^2y^2 = R_y$, and $R_x = 8xy^3 = P_z$, $\mathbf{F}$ is conservative. Thus, the work done between two points is independent of the path. From $\phi_x = 8xy^3z$ we obtain $\phi = 4x^2y^3z$ which is a potential function for $\mathbf{F}$. Then

$$W = \int_{(2,0,0)}^{(1,\sqrt{3},\pi/3)} \mathbf{F}\cdot d\mathbf{r} = 4x^2y^3z\Big|_{(2,0,0)}^{(1,\sqrt{3},\pi/3)} = 4\sqrt{3}\,\pi \quad \text{and} \quad W = \int_{(2,0,0)}^{(0,2,\pi/2)} \mathbf{F}\cdot d\mathbf{r} = 0.$$