Sec 9.8

5. $$\int_C z\,dx = \int_0^{\pi/2} t(-\sin t)\,dt \qquad \boxed{\text{Integration by parts}}$$

$$= (t\cos t - \sin t)\Big|_0^{\pi/2} = -1$$

$$\int_C z\,dy = \int_0^{\pi/2} t\cos t\,dt \qquad \boxed{\text{Integration by parts}}$$

$$= (t\sin t + \cos t)\Big|_0^{\pi/2} = \frac{\pi}{2} - 1$$

$$\int_C z\,dz = \int_0^{\pi/2} t\,dt = \frac{1}{2}t^2\Big|_0^{\pi/2} = \frac{\pi^2}{8}$$

$$\int_C z\,ds = \int_0^{\pi/2} t\sqrt{\sin^2 t + \cos^2 t + 1}\,dt = \sqrt{2}\int_0^{\pi/2} t\,dt = \frac{\pi^2\sqrt{2}}{8}$$

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10. From $(-1,2)$ to $(-1,0)$ we use y as a parameter with $x=-1$ and $dx=0$. From $(-1,0)$ to $(2,0)$ we use x as a parameter with $y=dy=0$. From $(2,0)$ to $(2,5)$ we use y as a parameter with $x=2$ and $dx=0$.

$$\int_C (2x+y)\,dx + xy\,dy = \int_2^0 (-1)\,y\,dy + \int_{-1}^2 2x\,dx + \int_0^5 2y\,dy = -\frac{1}{2}y^2\Big|_2^0 + x^2\Big|_{-1}^2 + y^2\Big|_0^5$$
$$= 2 + 3 + 25 = 30$$

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16. $$\int_C (-y^2)\,dx + xy\,dy = \int_0^2 (-t^6)\,2\,dt + \int_0^2 (2t)(t^3)3t^2\,dt = \int_0^2 4t^6\,dt = \frac{4}{7}t^7\Big|_0^2 = \frac{512}{7}$$

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22. From $(2,4)$ to $(0,4)$ we use x as a parameter with $y=4$ and $dy=0$. From $(0,4)$ to $(0,0)$ we use y as a parameter with $x=dx=0$. From $(0,0)$ to $(2,4)$ we use $y=2x$ and $dy=2\,dx$.

$$\oint_C x^2y^3\,dx - xy^2\,dy = \int_2^0 x^2(64)\,dx - \int_4^0 0\,dy + \int_0^2 x^2(8x^3)\,dx - \int_0^2 x(4x^2)2\,dx$$
$$= \frac{64}{3}x^3\Big|_2^0 + \frac{4}{3}x^6\Big|_0^2 - 2x^4\Big|_0^2 = -\frac{512}{3} + \frac{256}{3} - 32 = -\frac{352}{3}$$

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30. $\mathbf{F} = e^t\mathbf{i} + te^{t^3}\mathbf{j} + t^3e^{t^6}\mathbf{k}$; $\;d\mathbf{r} = (\mathbf{i} + 2t\mathbf{j} + 3t^2\mathbf{k})\,dt$;

$$\int_C \mathbf{F}\cdot d\mathbf{r} = \int_0^1 \left(e^t + 2t^2e^{t^3} + 3t^5e^{t^6}\right)dt = \left(e^t + \frac{2}{3}e^{t^3} + \frac{1}{2}e^{t^6}\right)\Big|_0^1 = \frac{13}{6}(e-1)$$