Sec 9.7

8. curl $\mathbf{F} = -2x^2\mathbf{i} + (10y - 18x^2)\mathbf{j} + (4xz - 10z)\mathbf{k}$; div $\mathbf{F} = 0$

14. curl $\mathbf{F} = (2xyz^3 + 3y)\mathbf{i} + (y\ln x - y^2z^3)\mathbf{j} + (2 - z\ln x)\mathbf{k}$; div $\mathbf{F} = \dfrac{yz}{x} - 3z + 3xy^2z^2$

23. $\mathbf{r}\times\mathbf{a} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x & y & z \\ a_1 & a_2 & a_3 \end{vmatrix} = (a_3y - a_2z)\mathbf{i} - (a_3x - a_1z)\mathbf{j} + (a_2x - a_1y)\mathbf{k}$; $\mathbf{r}\cdot\mathbf{r} = x^2 + y^2 + z^2$

$$\nabla\times[(\mathbf{r}\cdot\mathbf{r})\mathbf{a}] = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ (\mathbf{r}\cdot\mathbf{r})a_1 & (\mathbf{r}\cdot\mathbf{r})a_2 & (\mathbf{r}\cdot\mathbf{r})a_3 \end{vmatrix}$$
$$= (2ya_3 - 2za_2)\mathbf{i} - (2xa_3 - 2za_1)\mathbf{j} + (2xa_2 - 2ya_1)\mathbf{k} = 2(\mathbf{r}\times\mathbf{a})$$

30. Assuming continuous second partial derivatives,

$$\text{div (curl } \mathbf{F}) = \nabla\cdot[(R_y - Q_z)\mathbf{i} - (R_x - P_z)\mathbf{j} + (Q_x - P_y)\mathbf{k}]$$
$$= (R_{yx} - Q_{zx} - (R_{xy} - P_{zy}) + (Q_{xz} - P_{yz}) = 0.$$