King Fahd University of Petroleum and Minerals
Department of Mathematical Sciences
Math 301 Exam III
Semester I, 2006- (061)
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Serial NO:	**B**
ID:	
Name:	

Q	1	2	3	4	5	6	TOTAL
	(4 each) **39**	**22**	**15**	**15**	**20**	**22**	

Say a prayer & Good luck ☺

(1) **<u>(In part (a) – (g)),</u>**

Let $f(x)=\begin{cases}\cos x & 0<x<\dfrac{3\pi}{2}\\ \sin x & \dfrac{3\pi}{2}\le x<2\pi\end{cases}$, and let

$FS_1(x)$ = half-range cosine expansion of f(x).

$FS_2(x)$ = half-range sine expansion of f(x).

$FS_3(x)$ = half-range Fourier series expansion of f(x).

and

$\overline{FS}_1(x)$ = the periodic extension of $FS_1(x)$.

$\overline{FS}_2(x)$ = the periodic extension of $FS_2(x)$.

$\overline{FS}_3(x)$ = the periodic extension of $FS_3(x)$.

<u>TRUE or FALSE:</u>

(a)	$\overline{FS}_1(10\pi)=\overline{FS}_2(10\pi)$.	T	F
(b)	$\overline{FS}_2\left(\dfrac{19\pi}{2}\right)\neq\overline{FS}_3\left(\dfrac{27\pi}{2}\right)$.	T	F
(c)	$\overline{FS}_3(12\pi)\neq\overline{FS}_2(12\pi)$.	T	F
(d)	g(x) is a periodic function with period 4π where $g(x)=\overline{FS}_3(x)+\overline{FS}_2(x)$.	T	F
(e)	The graph of $FS_2(x)$ has spikes near the discontinuities at $x=\dfrac{3\pi}{2}$.	T	F
(f)	The coefficient of the term containing cosx in the series $FS_1(x)$ equals $\dfrac{2}{\pi}$.	T	F
(g)	The coefficient of the term containing cos(3x) in the series $FS_2(x)$ equals $\dfrac{3}{2\pi}$.	T	F

<u>TRUE or FALSE:</u>

(h)	The surface integral of the normal component of the curl of a conservative vector field **F** over a surface ***S*** is equal to zero.	T	F
(i)	For a two-dimensional vector field **F** in the plane z = 0, Stokes' theorem is the same as Green's theorem.	T	F

(2) The transverse displacement u(x,t) of a vibrating beam of length L is determined from a fourth-order partial differential equation

$$a^2 \frac{\partial^4 u}{\partial x^4} + \frac{\partial^2 u}{\partial t^2} = 0, \quad 0 < x < L, \quad t > 0.$$

If the beam is **simply supported,** as shown in Figure 13.12, the boundary and initial condition are

$$u(0,t) = 0, \qquad u(L,t) = 0, \quad t > 0$$

$$\left.\frac{\partial^2 u}{\partial x^2}\right|_{x=0} = 0, \qquad \left.\frac{\partial^2 u}{\partial x^2}\right|_{x=L} = 0, \quad t > 0$$

$$u(x,0) = f(x), \qquad \left.\frac{\partial u}{\partial t}\right|_{t=0} = g(x), \quad 0 < \mathrm{x} < \mathrm{L}.$$

Solve for u(x,t). [*Hint: For convenience use* λ^4 *instead of* λ^2 *when separating variables*.]

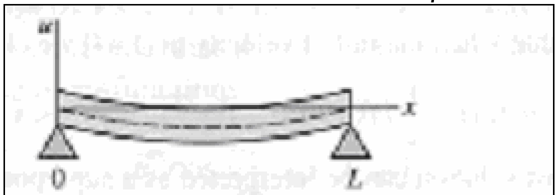

Figure 13.12 Simply supported beam

(3) Solve the heat equation subject to the condition: (Assume a rod of length 2π).

$$u(0,t) = 0 \quad , \quad u(2\pi,t) = 0$$

$$u(x,0) = \begin{cases} 1 & 0 < x < \pi \\ 0 & \pi \le x < 2\pi \end{cases}$$

Then find $\left.\frac{\partial u}{\partial x}\right|_{(t,x)=(0,0)}$

(4) Use separation of variable to find product solution of $\frac{\partial^2 u}{\partial x \partial y} = u$.[*Hint: For convenience use* $-\lambda$]

(5) Expand $f(x) = 2x^2 - 1, \ -1 < x < 1$, in a Fourier series.

(6) If $F = \frac{1}{3}x^3 i + \frac{1}{3}y^3 j + \frac{1}{3}z^3 k$, use the divergence theorem to evaluate $\iint_S (F \cdot n) ds$ where S is the surface of the region bounded by $x^2 + y^2 = 1$, z = 0, z = 1.