Name: KEY	**ID:** KEY

MATH-260 **Term-082** **QUIZ-2**

1) **Classify each differential equation as separable, linear, homogeneous, or Bernoulli. Some equations may be more than one kind. Do not solve**

	separable	linear	homog	Bernoulli
$y'=\dfrac{2xy+2x}{x^2+1}$	✓	✓ in y	×	✓ in x
$y'=\dfrac{3}{x(y+1)}$	✓	✓ in x	×	✓ in x
$y'=\dfrac{3}{y(x+1)}$	✓	✓ in x	×	✓ in y
$y'+(\sin x)y=(\cos x)y^{-3/4}$	×	×	×	✓

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2) **Find a general solution:** $yy''=3(y')^2$

We assume temporarily that y and y' are both nonnegative, and then point out at the end that this restriction is unnecessary.

The substitution $p=y'=dy/dx$, $y''=dp/dx=\frac{dp}{dy}\cdot\frac{dy}{dx}=p\frac{dp}{dy}$

give the first order DE

$$yp\frac{dp}{dy}=3p^2$$ divid by yp^2 yield

$\frac{1}{p}\,dp/dy=\frac{3}{y} \Rightarrow dp/p = 3\,dy/y$ (separable DE)

integrate both sides $\int dp/p = 3\int dy/y \Rightarrow \ln p = 3\ln y + c_0\ (p,y>0)$

$\Rightarrow p = y^3\cdot e^{c_0} \Rightarrow$ 2 $p=c_1y^3 \Rightarrow \frac{1}{p}=\frac{1}{c_1y^3} \Rightarrow \frac{dx}{dy}=\frac{1}{c_1y^3}$

$\Rightarrow c_1\,dx = y^{-3}dy \Rightarrow c_1x=-\frac{1}{2}y^{-2}+c_2 \Rightarrow y^{-2}=-2(c_1x-c_2)$

$\Rightarrow y^2=\frac{1}{\bar{c}_1x+\bar{c}_2} \Rightarrow$ 3 $\boxed{y=\pm 1/\sqrt{\bar{c}_1x+\bar{c}_2}}$ where $\bar{c}_1,\bar{c}_2$ real number (*)

(*) is the general solution for $yy''=3(y')^2$

3) Verify that the given differential equation is exact; then solve it. $y' = -\dfrac{e^x \sin y + \tan y}{e^x \cos y + x \sec^2 y}$

$$\frac{dy}{dx} = \frac{-(e^x \sin y + \tan y)}{(e^x \cos y + x \sec^2 y)} \Rightarrow (e^x \cos y + x \sec^2 y)$$

$$\Rightarrow (e^x \cos y + x \sec^2 y)\,dy = -(e^x \sin y + \tan y)\,dx$$

$$\Rightarrow \underbrace{(e^x \sin y + \tan y)}_{M}\,dx + \underbrace{(e^x \cos y + x \sec^2 y)}_{N}\,dy = 0$$

$$\frac{\partial M}{\partial y} = e^x \cos y + \sec^2 y \qquad \frac{\partial N}{\partial x} = e^x \cos y + \sec^2 y$$

Since $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \Rightarrow$ it is an exact DE. 2

$$f(x,y) = \int M\,dx + g(y) = \int (e^x \sin y + \tan y)\,dx + g(y) \longrightarrow (1)$$

$$= e^x \sin y + x \tan y + g(y) \quad 2$$

$$\frac{\partial f}{\partial y} = e^x \cos y + x \sec^2 y + g'(y) = N = e^x \cos y + x \sec^2 y$$

$$\Rightarrow g'(y) = 0 \Rightarrow g(y) = 0 \quad \text{--- } (2) \quad 2$$

(1) and (2) $\Rightarrow$ $f(x,y) = e^x \sin y + x \tan y$

The general solution:

$$\boxed{e^x \sin y + x \tan y = C}$$