

8) Write in  $L(y) = g(x)$  and factor  $L$ .

$$y''' + 4y'' + 3y' = x^2 \cos x - 3x$$

using  $y' = Dy, y'' = D^2y, y''' = D^3y$

$$D.E \Rightarrow (D^3 + 4D^2 + 3D)y = x^2 \cos x - 3x$$

$$\Rightarrow D(D^2 + 4D + 3D)y = x^2 \cos x - 3x$$

$$\Rightarrow D(D+1)(D+3)y = x^2 \cos x - 3x$$

Additional Note: Annihilator for R.H.S is  $D^2(D^2+1)^3$

13) Verify that  $y = e^{2x} + 3e^{-5x}$  has annihilator is  $(D-2)(D+5)$

$$(D-2)(D+5)(e^{2x} + 3e^{-5x})$$

$$= (D-2)[(D+5)e^{2x} + 3(D+5)e^{-5x}]$$

$$= (D-2)[2e^{2x} + 5e^{2x} + 3(-5e^{-5x} + 5e^{-5x})]$$

$$= 2 \cdot 2 e^{2x} - 4 e^{2x} + 5(2e^{2x} - 2e^{2x})$$

$$= 0. \text{ Hence the result.}$$

22) Find annihilator.

$8x - \sin x + 10 \cos 5x$   
 annihilator for  $8x$  :  $D^2$   
 annihilator for  $-\sin x$  :  $(D^2+1)$   
 annihilator for  $10 \cos 5x$  :  $(D^2+25)$   
 Thus required annihilator is  $D^2(D^2+1)(D^2+25)$ .

24) Find annihilator:  $(2 - e^x)^2 = 4 - 2e^x + e^{2x}$

The annihilator is therefore  $D(D-1)(D-2)$ .

41)  $y''' + y'' = 8x^2$  — (1)

Step 1 Homogeneous D.E  
 $m^3 + m^2 = 0 \Rightarrow m^2(m+1) = 0$   
 $\Rightarrow m = 0, 0, -1$

$$y_c = C_1 + C_2 x + C_3 e^{-x}$$

Step 2: Annihilator of R.H.S is  $D^3$

Apply to D.E  
 $D^3(D^3 + D^2)y = D^3(8x^2) = 0$   
 Auxiliary eqn:  $m^3(m^3 + m^2) = 0$   
 $m = 0, 0, 0, 0, 0, -1$

$$y = \underbrace{A + Bx + Cx^2 + Dx^3}_{y_c} + \underbrace{Ex^4}_{y_p}$$

Cancel out terms common with  $y_c$

$$y_p = Cx^2 + Dx^3 + Ex^4$$

$$y_p' = 2Cx + 3Dx^2 + 4Ex^3$$

$$y_p'' = 2C + 6Dx + 12Ex^2$$

$$y_p''' = 6D + 24Ex$$

D.E (1)  $\Rightarrow$   
 $6D + 24Ex + 2C + 6Dx + 12Ex^2 = 8x^2$

Compare coefficients of like terms

1:  $6D + 2C = 0$  — (a)  
 $x$ :  $6D + 24E = 0$  — (b)  $\Rightarrow 6D = -24E$   
 $x^2$ :  $12E = 8$  — (c)  
 $\Rightarrow E = \frac{8}{12} \Rightarrow E = \frac{2}{3}$   
 From (b)  $\Rightarrow D = -\frac{24E}{6} = -\frac{24 \times \frac{2}{3}}{6} = -\frac{8}{3}$   
 From (a)  $\Rightarrow C = 8$

Hence  $y_p = \frac{2}{3}x^4 - \frac{8}{3}x^3 + 8x^2$

Step 3  $y = y_c + y_p$  so

$$y = C_1 + C_2 x + C_3 e^{-x} + \frac{2}{3}x^4 - \frac{8}{3}x^3 + 8x^2$$

(4.5 contd)

48)  $y'' + 4y = 4\cos x + 3\sin x - 8$

Step 1 Homogeneous D.E:  $y'' + 4y = 0$ 

Auxiliary Eqn:  $m^2 + 4 = 0$

$\Rightarrow m = \pm 2i$

$y_c = C_1 \cos 2x + C_2 \sin 2x$

Step 2: Annihilator:  $D(D^2 + 1)$ 

Apply on D.E

$$D(D^2 + 1)(D^2 + 4)y = D(D^2 + 1)[4\cos x + 3\sin x - 8] = 0$$

Auxiliary eqn is

$m(m^2 + 1)(m^2 + 4) = 0$

$m = 0, \pm i, \pm 2i$

$y = A + B\cos x + C\sin x + D\cos 2x + E\sin 2x$

Cancel terms contained in  $y_c$ :

Thus  $y_p = A + B\cos x + C\sin x$

$y_p' = -B\sin x + C\cos x$

$y_p'' = -B\cos x - C\sin x$

D.E  $\Rightarrow -B\cos x - C\sin x + 4A + 4B\cos x + 4C\sin x = 4\cos x + 3\sin x - 8$

$+4C\sin x = 4\cos x + 3\sin x - 8$

Compare coefficients of like terms:

1:  $4A = -8 \Rightarrow A = -2$

$\cos x: -B + 4B = 4 \Rightarrow B = \frac{4}{3}$

$\sin x: -C + 4C = 3 \Rightarrow C = 1$

Thus  $y_p = -2 + \frac{4}{3}\cos x + \sin x$

Step 3:  $y = y_c + y_p$ 

$y = C_1 \cos 2x + C_2 \sin 2x - 2 + \frac{4}{3}\cos x + \sin x$

64)  $y^{(4)} - 4y'' = 5x^2 - e^{2x}$

Step 1: Homogeneous DE:  $y^{(4)} - 4y'' = 0$ 

Auxiliary Eqn:  $m^4 - 4m^2 = 0$

$m = 0, 0, -2, 2$

$y_c = C_1 + C_2 x + C_3 e^{-2x} + C_4 e^{2x}$

Step 2: Annihilator of R.H.S

$D^3(D-2)$

Apply on D.E

$D^3(D-2)[D^4 - 4D^2]y = D^3(D-2)[5x^2 - e^{2x}] = 0$

Auxiliary equation is

$m^3(m-2)(m^4 - 4m^2) = 0$

$m = \underbrace{0, 0, 0, 0, 0}_{5 \text{ values}}, m = 2, 2, -2$

$y = \underbrace{A + Bx}_{y_c} + Cx^2 + Dx^3 + Ex^4 + \underbrace{Fe^{2x}}_{y_c} + Gx^2 e^{2x} + H e^{-2x}$

Cancel terms common with  $y_c$ 

$y_p = Cx^2 + Dx^3 + Ex^4 + Gx^2 e^{2x}$

$y_p' = 2Cx + 3Dx^2 + 4Ex^3 + Ge^{2x} + 2Gxe^{2x}$

$y_p'' = 2C + 6Dx + 12Ex^2 + 2Ge^{2x} + 2Ge^{2x} + 4Gxe^{2x} = 2C + 6Dx + 12Ex^2 + 4Ge^{2x} + 4Gxe^{2x}$

$y_p''' = 6D + 24Ex + 8Ge^{2x} + 4Ge^{2x} + 8Gxe^{2x}$

$= 6D + 24Ex + 12Ge^{2x} + 8Gxe^{2x}$

$y_p^{(4)} = 24E + 24Ge^{2x} + 8Ge^{2x} + 16Gxe^{2x}$

$= 24E + 32Ge^{2x} + 16Gxe^{2x}$

Put in D.E

$24E + 32Ge^{2x} + 16Gxe^{2x} - 8C - 24Dx - 48Ex^2 - 16Ge^{2x} - 16Gxe^{2x} = 5x^2 - e^{2x}$

Compare Coefficients

1:  $24E - 8C = 0 \Rightarrow C = 3E$

$x: -24D = 0 \Rightarrow D = 0$

$x^2: -48E = 5 \Rightarrow E = -\frac{5}{48}$

$e^{2x}: 32G - 16G = -1 \Rightarrow G = -\frac{1}{16}$

$xe^{2x}: 16G - 16G = 0$

$C = 3E = -\frac{5}{16}$

$y_p = -\frac{5}{16}x^2 - \frac{5}{48}x^4 - \frac{1}{16}xe^{2x}$

Step 3  $y = y_c + y_p$ 

$= C_1 + C_2 x + C_3 e^{-2x} + C_4 e^{2x} - \frac{5}{16}x^2 - \frac{5}{48}x^4 - \frac{1}{16}xe^{2x}$