Math 202

4.2

1) $y'' - 6y' + 9y = 0, \quad y_1 = e^{-3x}$

$$y_2 = y_1 \int \frac{e^{-\int P(x)dx}}{y_1^2} dx$$

$$= e^{-3x} \int \frac{e^{-\int 6 dx}}{e^{-6x}} dx =$$

$$e^{-3x} \int \frac{e^{-6x}}{e^{-6x}} dx = e^{-3x} \int dx$$

$$= x e^{-3x}.$$

Thus $y_2 = x e^{-3x}$.

3) $y'' + 16y = 0, \quad y_1 = \cos 4x$.

Here $P(x) = 0$.

$$y_2 = \cos 4x \int \frac{e^{-\int 0 dx}}{\cos^2 4x} dx$$

$$= \cos 4x \int \sec^2 4x \, dx$$

$$= \cos 4x \left(\frac{1}{4} \tan 4x\right) = \frac{1}{4} \sin 4x$$

Thus we can take

$y_2 = \sin 4x$.

(which is 4 times y_2 we get)

12) $4x^2 y'' + y = 0; \quad y_1 = x^{1/2} \ln x$

Here $P(x) = 0$

$$y_2 = x^{1/2} \ln x \int \frac{1}{x (\ln x)^2} dx$$

In the integral, put $\ln x = t$

$\frac{1}{x} dx = dt$

$$y_2 = x^{1/2} \ln x \int t^{-2} dt = x^{1/2} \ln x \left[-t^{-1}\right]$$

$$= -x^{1/2} \ln x \cdot \frac{1}{\ln x} = -x^{1/2}.$$

14) Similar to 12 above.

19) $y'' - 3y' + 2y = 5e^{3x}, \; y_1 = e^{x}$.

I The homogeneous equation is

$y'' - 3y' + 2y = 0. \quad , y_1 = e^{x}$

$$y_2 = e^{x} \int \frac{e^{+\int 3 dx}}{e^{2x}} dx$$

$$= e^{x} \int \frac{e^{+3x}}{e^{2x}} dx = e^{x} \int e^{x} dx$$

$$= e^{x}(e^{x}) = e^{2x}$$

Thus we take $y_2 = e^{2x}$

$y_c = C_1 e^{x} + C_2 e^{2x}$

II y_p : Assume $y_p = A e^{3x}$

$y_p' = 3A e^{3x}$

$y_p'' = 9A e^{3x}$

Given D.E gives

$9Ae^{3x} - 9Ae^{3x} + 2Ae^{3x} = 5e^{3x}$

Comparing both sides

$2A = 5 \Rightarrow A = \frac{5}{2}$.

Thus $y_p = \frac{5}{2} e^{3x}$.

General solution

$y = y_c + y_p$ so

$$y = c_1 e^{x} + c_2 e^{2x} + \frac{5}{2} e^{3x}.$$