

(1.2)

$$2) \quad y = \frac{1}{1+C_1 e^{-x}}$$

Given $y(-1) = 2$ i.e.

At $x = -1, y = 2$.

This gives $2 = \frac{1}{1+C_1 e}$

$$\Rightarrow 2 + 2C_1 e = 1 \Rightarrow C_1 = -\frac{1}{2e}$$

Thus IVP has solution

$$y = \frac{1}{1 - \frac{1}{2e} e^{-x}} = \frac{2}{2 - e^{-(x+1)}}$$

unique solution may exist in $y < -1$ or $y > -1$.

$$27) \quad y' = \sqrt{y^2 - 9} = f(x, y)$$

$$f(x, y) = \sqrt{y^2 - 9}$$

$$\frac{\partial f}{\partial y} = \frac{y}{2} \cdot 2 (y^2 - 9)^{-1/2} = \frac{y}{\sqrt{y^2 - 9}}$$

Thus region should ~~be~~ not include $y = \pm 3$ or $-3 < y < 3$.

The given point $(2, -3)$.

Any rectangle containing $(2, -3)$ will include $y = -3$, thus the condition of the theorem will be violated. No unique solution is guaranteed.

$$12) \quad y = c_1 e^x + c_2 e^{-x}, \quad y' = c_1 e^x - c_2 e^{-x}$$

$$y(1) = 0 \Rightarrow c_1 e + c_2 e^{-1} = 0 \quad (1)$$

$$y'(1) = e \Rightarrow c_1 e - c_2 e^{-1} = e \quad (2)$$

Solving (1) and (2) for c_1, c_2

$$(1) \Rightarrow c_1 e = -c_2 e^{-1} \Rightarrow c_1 = -c_2 e^{-2}$$

Put in (2)

$$-c_2 e^{-2} e - c_2 e^{-1} = e \Rightarrow c_2 = -\frac{e^2}{2}$$

$$\text{Thus } c_1 = -\frac{e^2}{2} e^{-2} = \frac{1}{2}$$

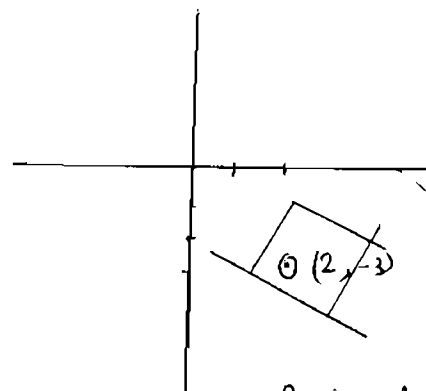
$$\text{So, } y = \frac{1}{2} e^x - \frac{e^2}{2} e^{-x}$$

$$22) \quad (1+y^3) y' = x^2$$

$$y' = \frac{x^2}{1+y^3} = f(x, y)$$

$$\frac{\partial f}{\partial y} = \frac{-3y^2 x^2}{(1+y^3)^2}$$

$f(x, y)$ and $\frac{\partial f}{\partial y}$ are not continuous in domain containing $y = -1$. Thus



Rectangle containing $(2, -3)$.