

**King Fahd University of Petroleum and Minerals**  
**Department of Mathematics and Statistics**  
**Math 260 Final Exam**  
**Semester II, 2010- (092)**

<b>Name:</b>			
<b>ID:</b>		<b>Serial no:</b>	

**Section**

<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>
7:00-7:50	8:00-8:50	9:00-9:50	10:00-10:50
Dr. Fairag	Dr. Fairag	Dr. Laradji	Dr. Fairag

Q	Your Points	Points
1		<b>20</b>
2		<b>20</b>
3		<b>20</b>
4		<b>20</b>
5		<b>20</b>
6		<b>20</b>
7		<b>20</b>
8		<b>10</b>
9		<b>10</b>
10		<b>10</b>
11		<b>10</b>
12		<b>10</b>
13		<b>10</b>
Total		<b>200</b>

( ) Say Bismillah & Good luck ( )

(1) Solve the system  $X' = AX$

where  $A = \begin{bmatrix} 6 & -1 \\ 5 & 2 \end{bmatrix}$ . ( $\lambda = 4 \pm i$  are eigenvalues of A)

(2) Solve the system  $X' = AX$

where  $A = \begin{bmatrix} -2 & 0 & 0 \\ 4 & 4 & 6 \\ -4 & -6 & -8 \end{bmatrix}$ . (given that  $\lambda = -2$  is an eigenvalue of multiplicity 3)

(3) Solve the initial value problem

$$y''' + 4y' = 0$$

$$y(0) = -1, \quad y'(0) = 2, \quad y''(0) = 4$$

(4) Find a particular solution of

$$y'' + 2y' - 3y = 3 - 16xe^x$$

(5) Solve the differential equation

$$(x + 2 \tan^{-1} y)dx + \left(\frac{2x + y}{1 + y^2}\right)dy = 0$$

(6) Solve the differential equation

$$y'' + (y')^2 + 1 = 0$$

(7) Verify that

$$X_1 = e^{-2t} \begin{bmatrix} 3 \\ -2 \\ 2 \end{bmatrix}, \quad X_2 = e^t \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}, \quad X_3 = e^{3t} \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$$

are linearly independent solutions of the system  $X' = AX$

where  $A = \begin{bmatrix} -8 & -11 & -2 \\ 6 & 9 & 2 \\ -6 & -6 & 1 \end{bmatrix}$ . Find a particular solution of the system

satisfying the initial conditions  $x_1(0) = 1, x_2(0) = 2, x_3(0) = 3$

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(8) The rank of the matrix

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 \\ 3 & 3 & 3 & 3 \\ 4 & 4 & 4 & 4 \end{bmatrix}$$

equals

- (a) 1
- (b) 2
- (c) 3
- (d) 4

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(9) Compute the determinant

$$\begin{vmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{vmatrix} =$$

- (e) -2
- (f) -1
- (g) 0
- (h) 1
- (i) 2

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(10) The differential equation  $3y^2y' + y^3 = e^{-x}$  is

- (a) Separable      (b) Linear      (c) Bernoulli      (e) None of the above

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(11) The matrix  $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}$  is

- (a) diagonalizable      (b) not diagonalizable

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(12) The set of vectors  $\left\{ \begin{bmatrix} 1 \\ 6 \\ -13 \end{bmatrix}, \begin{bmatrix} 2e^{3t} \\ 3e^{3t} \\ -2e^{3t} \end{bmatrix}, \begin{bmatrix} -e^{-4t} \\ 2e^{-4t} \\ e^{-4t} \end{bmatrix} \right\}$  is

- (a) linearly independent      (b) linearly dependent

(13) Let  $A$  be an  $n \times n$  matrix such that  $A^3 = 0$ . show that  $\lambda = 0$  is an eigenvalue of  $A$ .