

**King Fahd University of Petroleum and Minerals**  
**Department of Mathematics and Statistics**  
**Math 260 Exam-II**  
**Semester II, 2010- (092)**

<b>Name:</b>			
<b>ID:</b>		<b>Serial no:</b>	

**Section**

<b>1</b>
<b>7:00-7:50</b>
<b>Dr. Fairag</b>

<b>2</b>
<b>8:00-8:50</b>
<b>Dr. Fairag</b>

<b>3</b>
<b>9:00-9:50</b>
<b>Dr. Laradji</b>

<b>4</b>
<b>10:00-10:50</b>
<b>Dr. Fairag</b>

Q	FORM: <b>A</b>	Points
1		<b>10</b>
2		<b>10</b>
3		<b>10</b>
4		<b>10</b>
5		<b>13</b>
6		<b>10</b>
7		<b>14</b>
8		<b>10</b>
9		<b>13</b>
Total		<b>100</b>

 Say Bismillah & Good luck 

(1) Use Cayley-Hamilton theorem to compute  $A^4 - 3A^3$

where  $A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 1 & -1 \\ -2 & -3 & 1 \end{bmatrix}$ . **(show all your work)**

(2) Find a basis for the column space of the matrix

$$A = \begin{bmatrix} 1 & -3 & 4 & -2 & 5 & 4 \\ 2 & -6 & 9 & -1 & 8 & 2 \\ 2 & -6 & 9 & -1 & 9 & 7 \\ -1 & 3 & -4 & 2 & -5 & -4 \end{bmatrix}$$

**(show all your work)**

(3) Find the rank of the matrix

$$A = \begin{bmatrix} 1 & 1 & -2 & 0 & 0 \\ 1 & 2 & 3 & 6 & 7 \\ 2 & 1 & 3 & 6 & 5 \end{bmatrix}$$

**(show all your work)**

(4) Determine whether or not the matrix

$$A = \begin{bmatrix} -2 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix}$$

is diagonalizable. **(show all your work)**

(5) The characteristic equation of the matrix  $C = \begin{bmatrix} 3 & 1 & -2 \\ -1 & 0 & 5 \\ -1 & -1 & 4 \end{bmatrix}$  is  $(2-x)^2(3-x)$ .

Find the Jordan form of the matrix  $C$ . (show all your work)

(6) Given that the matrix  $A = \begin{bmatrix} 5 & -4 & 4 \\ 12 & -11 & 12 \\ 4 & -4 & 5 \end{bmatrix}$  is diagonalizable and has eigenvalues  $\lambda = -3, 1, 1$ .

Find a diagonalizing matrix  $P$  and a diagonal matrix  $D$  such that  $D = P^{-1}AP$ .  
**(show all your work)**

(7) True or False.

[a]	The order of the differential equation $\left(\frac{d^3 y}{dx^3}\right)^5 + 3\frac{d^2 y}{dx^2} - 9\left(\frac{d^4 y}{dx^4}\right)^2 - y = 0$ is 5.	(T) (F)
[b]	The equation $\left(\frac{\partial u}{\partial x}\right)^2 + \frac{\partial u}{\partial y} = y \sin x$ is a partial differential equation.	(T) (F)
[c]	$y(x) = 0$ is a singular solution for the differential equation $y^{-1/2} \frac{dy}{dx} = x$ .	(T) (F)
[d]	$y(x) = 0$ is a singular solution for $y' = xy^{1/2}$ .	(T) (F)
[e]	Let A be $4 \times 4$ matrix. A has eigenvalues $\lambda = 2, 2, 5, 7$ , then $\text{Rank}(A) \neq 4$ .	(T) (F)
[f]	$y(x) = x + \ln x$ is a particular solution of the differential equation $x^2 y'' + xy' - y = \ln x$ .	(T) (F)
[g]	Let A be $4 \times 4$ matrix with characteristic polynomial $p(\lambda) = \lambda(\lambda - 2)^3$ , then $\det(A) = 0$ .	(T) (F)



(8) Find the eigenvalues of

$$A = \begin{bmatrix} 0 & 2 & 1 \\ 3 & -1 & 1 \\ 1 & -1 & 1 \end{bmatrix} . \text{ (show all your work)}$$

(9) Solve the separable differential equation

$$2(y^2 + 2y) \frac{dy}{dx} = (3x^2 - 4)(y^2 - 1)(y + 2) \quad . \quad (\text{show all your work})$$

