King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics
Math 260 Exam-1
Semester II, 2010- (092)

Name:			
ID:		**Serial no:**	

Section

1	2	3	4
7:00-7:50	8:00-8:50	9:00-9:50	10:00-10:50
Dr. Fairag	Dr. Fairag	Dr. Laradji	Dr. Fairag

Q	FORM: **A**	Points
1		**12**
2		**9**
3		**10**
4		**10**
5		**9**
6		**22**
7		**7**
8		**7**
9		**7**
10		**7**
Total		**100**

☺ **Say Bismillah & Good luck** ☺

(1) Consider the homogeneous linear system of equations

$$\begin{aligned} x_1 + x_2 + 3x_3 + 3x_4 \quad &= 0 \\ -x_1 \quad - 2x_3 - x_4 + x_5 &= 0 \\ 2x_1 + 3x_2 + 7x_3 + 8x_4 + x_5 &= 0 \end{aligned}$$

(a) Find the solution space W.

(b) Find a basis for W.

(c) Find dim(W).

(2) Given that: $u = \begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix}$, $v = \begin{bmatrix} 4 \\ 1 \\ -1 \end{bmatrix}$, $w = \begin{bmatrix} 1 \\ 1 \\ 5 \end{bmatrix}$ and $t = \begin{bmatrix} 2 \\ 5 \\ 3 \end{bmatrix}$,

Express u as a linear combination of the vectors v, w, t.

(3) Compute A^{-1} if $A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$.

(4) Let W be the set of all vectors $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ in R^3 such that $x_3 = -2x_1$. Is W a subspace of R^3? (either prove it is or show that it is not a subspace of R^3).

(5) Determine for what values of k the set **S** is linearly independent in R^4.

$$S = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 100 \\ 1 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -9 \\ 1 \\ 1 \\ k \end{bmatrix} \right\}$$

(6) True or False.

[1]	The inverse of any invertible matrix is unique.	(T) (F)
[2]	The determinant of a matrix is equal to the determinant of its transpose.	(T) (F)
[3]	Any subset of a linearly independent set $S = \{v_1, v_2, \cdots, v_k\}$ is a linearly independent set of vectors.	(T) (F)
[4]	Let A be an nxn matrix. If **A** has nonzero determinant then **A** is row equivalent to the identity matrix **I**.	(T) (F)
[5]	Any set of more than n vectors in R^n is linearly dependent	(T) (F)
[6]	The vectors $v_1, v_2, \cdots, v_k$ are linearly dependent if and only if one of them is a linear combination of the others	(T) (F)
[7]	If u, v and w in R^3 are linearly independent, then they constitute a basis for R^3.	(T) (F)
[8]	The set $S = \left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \end{bmatrix} \right\}$ is a basis for R^2	(T) (F)
[9]	The set $S = \left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \end{bmatrix} \right\}$ is a basis for R^2	(T) (F)
[10]	The set $S = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 5 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 0 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \\ 1 \\ 9 \end{bmatrix} \right\}$ is linearly independent in R^4	(T) (F)
[11]	The set $S = \left\{ \begin{bmatrix} 6 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \end{bmatrix} \right\}$ spans R^2	(T) (F)

c

(7) Which of the following subsets of R^4 is linearly independent?

(a) $\left\{\begin{bmatrix}1\\2\\3\\4\end{bmatrix},\begin{bmatrix}2\\-1\\1\\-2\end{bmatrix},\begin{bmatrix}3\\1\\4\\2\end{bmatrix},\begin{bmatrix}3\\4\\1\\2\end{bmatrix}\right\}$

(b) $\left\{\begin{bmatrix}1\\2\\3\\4\end{bmatrix},\begin{bmatrix}-1\\2\\-3\\4\end{bmatrix},\begin{bmatrix}1\\-2\\-3\\4\end{bmatrix},\begin{bmatrix}0\\0\\0\\0\end{bmatrix}\right\}$

(c) $\left\{\begin{bmatrix}1\\2\\3\\4\end{bmatrix},\begin{bmatrix}2\\1\\4\\3\end{bmatrix},\begin{bmatrix}3\\1\\4\\2\end{bmatrix},\begin{bmatrix}3\\4\\1\\2\end{bmatrix},\begin{bmatrix}1\\3\\2\\4\end{bmatrix}\right\}$

(d) $\left\{\begin{bmatrix}1\\2\\3\\4\end{bmatrix},\begin{bmatrix}2\\1\\-2\\0\end{bmatrix},\begin{bmatrix}1\\-1\\-5\\-4\end{bmatrix},\begin{bmatrix}4\\2\\-4\\0\end{bmatrix}\right\}$

(e) $\left\{\begin{bmatrix}1\\2\\3\\4\end{bmatrix},\begin{bmatrix}2\\3\\4\\0\end{bmatrix},\begin{bmatrix}3\\4\\0\\0\end{bmatrix},\begin{bmatrix}2\\0\\0\\0\end{bmatrix}\right\}$

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(8) The value of k for which the system

$$\begin{aligned} x+y+2z&=1\\ x-y+z&=2\\ -x-2y+z&=3\\ 2x-y+2z&=k \end{aligned}$$

has a unique solution is

(a) 5
(b) 4
(c) 3
(d) 2

(e) 1

(9) The augmented coefficient matrix of a linear system of equations has reduced row echelon form

$$\left[\begin{array}{cccccc|c} 1 & 2 & 0 & 0 & -3 & 0 & 0 \\ 0 & 0 & 1 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 1 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right].$$

Then the number of free variables is

(a) 0
(b) 1
(c) 2
(d) 3
(e) 4

(10) $$\begin{vmatrix} 1 & -2 & 3 & 1 \\ 5 & -9 & 6 & 3 \\ -1 & 2 & -6 & -2 \\ 2 & 8 & 6 & 1 \end{vmatrix} =$$

(a) -39
(b) -36
(c) 0
(d) 39
(e) 36

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