Name: KEY	ID:

MATH-260 Term-082 QUIZ-7

1) Suppose that A is an $n \times n$ matrix and that k is a (constant) scalar. Show that the set of all vectors x such that $Ax = kx$ is subspace of R^n.

Let W = the set of all vectors x such that $Ax = kx$

let $x_1, x_2 \in W \Rightarrow Ax_1 = kx_1$ ① and $Ax_2 = kx_2$ ②

let $z = x_1 + x_2$, then $Az = A(x_1 + x_2) = Ax_1 + Ax_2$

①,② $\Rightarrow$ $= kx_1 + kx_2 = k(x_1 + x_2) = kz$

Hence $z \in W$.

let $x \in W \Rightarrow Ax = kx$

let $z = cx \Rightarrow Az = A(cx) = cAx = cz \Rightarrow z \in W$

Hence, W is a subspace

2) Find a basis for the solution space of the given homogeneous linear system.

$$x_1 - 2x_2 + 3x_3 = 0$$
$$2x_1 - 3x_2 - x_3 = 0$$

The augmented matrix $\left[\begin{array}{ccc|c} 1 & -2 & 3 & 0 \\ 2 & -3 & -1 & 0 \end{array}\right] \xrightarrow{-2R_1 + R_2}$

$\left[\begin{array}{ccc|c} 1 & -2 & 3 & 0 \\ 0 & 1 & -7 & 0 \end{array}\right] \xrightarrow{2R_2 + R_1} \left[\begin{array}{ccc|c} 1 & 0 & -11 & 0 \\ 0 & 1 & -7 & 0 \end{array}\right] = [E \mid 0]$

Leading variables: x_1, x_2
Free variables: $x_3 = t$

$\Rightarrow$ $x_1 = 11x_3 = 11t$, $x_2 = 7x_3 = 7t$

The solution : $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 11t \\ 7t \\ t \end{bmatrix}$

$t = 1 \Rightarrow v_1 = \begin{bmatrix} 11 \\ 7 \\ 1 \end{bmatrix}$

$\{v_1\}$ forms basis for the solution space.

3) Find the general solution. $4y''+4y'+y=0$

charc. eqn : $4r^2+4r+1=0$

$(2r+1)^2=0$

roots : $-\frac{1}{2}$, $-\frac{1}{2}$

Sols : $e^{-\frac{1}{2}x}$, $x e^{-\frac{1}{2}x}$

The general solution : $y=c_1 e^{-\frac{1}{2}x}+c_2 x e^{-\frac{1}{2}x}$

4) Use the Wronskian to prove that th given functions are linearly independent.

$$f(x)=x,\ g(x)=xe^x,\ h(x)=x^2e^x$$

$$W=\begin{vmatrix} x & xe^x & x^2e^x \\ 1 & e^x+xe^x & 2xe^x+x^2e^x \\ 0 & 2e^x+xe^x & 2e^x+4xe^x+x^2e^x \end{vmatrix}$$

$$\xlongequal{-xR_2+R_1}\begin{vmatrix} 0 & -x^2e^x & -x^2e^x-x^3e^x \\ 1 & e^x+xe^x & 2xe^x+x^2e^x \\ 0 & 2e^x+xe^x & 2e^x+4xe^x+x^2e^x \end{vmatrix}$$

expand along the first column.

$$=(-1)(1)\begin{vmatrix} -x^2e^x & -x^2e^x-x^3e^x \\ 2e^x+xe^x & 2e^x+4xe^x+x^2e^x \end{vmatrix}=-\begin{vmatrix} x^2e^x & (x^2+x^3)e^x \\ (2+x)e^x & (2+4x+x^2)e^x \end{vmatrix}$$

$$=-e^{2x}\begin{vmatrix} x^2 & x^2+x^3 \\ 2+x & 2+4x+x^2 \end{vmatrix}=-e^{2x}\left[\left(\cancel{2x^2}+4x^3+\cancel{x^4}\right)-\left(\cancel{2x^2}+2x^3+x^3+\cancel{x^4}\right)\right]$$

$$=-e^{2x}(x^3)=-x^3e^{2x}\neq 0 \Rightarrow \text{lin. indep.}$$