Name: KEY	ID: KEY	serial:

MATH-260 Term-082 QUIZ-4

1) If $A=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then show that: $A^2=(a+d)A-(ad-bc)I$, Where I denotes the 2 X 2 identity matrix.

$$LHS = A^2 = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a^2+bc & ab+bd \\ ca+dc & cb+d^2 \end{bmatrix}$$

3

$$RHS = (a+d)A-(ad-bc)I = (a+d)\begin{bmatrix} a & b \\ c & d \end{bmatrix} - \begin{bmatrix} ad-bc & 0 \\ 0 & ad-bc \end{bmatrix}$$

$$= \begin{bmatrix} a^2+ad & ab+db \\ ac+cd & ad+d^2 \end{bmatrix} - \begin{bmatrix} ad-bc & 0 \\ 0 & ad-bc \end{bmatrix} = \begin{bmatrix} a^2+bc & ab+db \\ ca+dc & cb+d^2 \end{bmatrix}$$

$= LHS$

2) Use the inverse to solve the linear system $\begin{bmatrix} 1 & 1 & 5 \\ 1 & 4 & 13 \\ 3 & 2 & 12 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 5 & 1 & 0 & 0 \\ 1 & 4 & 13 & 0 & 1 & 0 \\ 3 & 2 & 12 & 0 & 0 & 1 \end{array}\right] \xrightarrow[-3R_1+R_3]{-R_1+R_2} \left[\begin{array}{ccc|ccc} 1 & 1 & 5 & 1 & 0 & 0 \\ 0 & 3 & 8 & -1 & 1 & 0 \\ 0 & -1 & -3 & -3 & 0 & 1 \end{array}\right] \xrightarrow{R_2 \leftrightarrow R_3}$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 5 & 1 & 0 & 0 \\ 0 & -1 & -3 & -3 & 0 & 1 \\ 0 & 3 & 8 & -1 & 1 & 0 \end{array}\right] \xrightarrow{-R_2} \left[\begin{array}{ccc|ccc} 1 & 1 & 5 & 1 & 0 & 0 \\ 0 & 1 & 3 & 3 & 0 & -1 \\ 0 & 3 & 8 & -1 & 1 & 0 \end{array}\right] \xrightarrow{-3R_2+R_3}$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 5 & 1 & 0 & 0 \\ 0 & 1 & 3 & 3 & 0 & -1 \\ 0 & 0 & -1 & -10 & 1 & 3 \end{array}\right] \xrightarrow{-R_3} \left[\begin{array}{ccc|ccc} 1 & 1 & 5 & 1 & 0 & 0 \\ 0 & 1 & 3 & 3 & 0 & -1 \\ 0 & 0 & 1 & 10 & -1 & -3 \end{array}\right] \xrightarrow[-5R_3+R_1]{-3R_3+R_2}$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & -49 & 5 & 15 \\ 0 & 1 & 0 & -27 & 3 & 8 \\ 0 & 0 & 1 & 10 & -1 & -3 \end{array}\right] \xrightarrow{-R_2+R_1} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -22 & 2 & 7 \\ 0 & 1 & 0 & -27 & 3 & 8 \\ 0 & 0 & 1 & 10 & -1 & -3 \end{array}\right]$$

$$\Rightarrow A^{-1} = \begin{bmatrix} -22 & 2 & 7 \\ -27 & 3 & 8 \\ 10 & -1 & -3 \end{bmatrix} \; 3 \quad \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = A^{-1}\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

5

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -22 \\ -27 \\ 10 \end{bmatrix} \; 2$$

Note:
(1) $R_1 - R_2$ is not row operation
(2) How to find A^{-1} by seq. of row operations

3) Evaluate $\begin{vmatrix} 1 & 2 & 1 & -1 & 10 \\ 2 & 1 & 3 & 3 & 9 \\ 0 & 1 & -2 & 3 & 1 \\ -1 & 4 & -2 & 4 & -20 \\ 0 & 0 & 0 & 0 & 1 \end{vmatrix}$ expand along the fifth row

$$= (+1)\begin{vmatrix} 1 & 2 & 1 & -1 \\ 2 & 1 & 3 & 3 \\ 0 & 1 & -2 & 3 \\ -1 & 4 & -2 & 4 \end{vmatrix} \underset{-2R_1+R_2}{\overset{R_1+R_4}{=}}$$

$$\begin{vmatrix} 1 & 2 & 1 & -1 \\ 0 & -3 & 1 & 5 \\ 0 & 1 & -2 & 3 \\ 0 & 6 & -1 & 3 \end{vmatrix} = (1)\begin{vmatrix} -3 & 1 & 5 \\ 1 & -2 & 3 \\ 6 & -1 & 3 \end{vmatrix} \underset{-6R_2+R_3}{\overset{3R_2+R_1}{=}}$$

$$\begin{vmatrix} 0 & -5 & 14 \\ 1 & -2 & 3 \\ 0 & 11 & -15 \end{vmatrix} = (-)(1)\begin{vmatrix} -5 & 14 \\ 11 & -15 \end{vmatrix} = (-1)\left[(-5)(-15)-(11)(14)\right]$$

$$= -\left[75-154\right] = 79$$

4

4) The square matrix A is called orthogonal provided that $A^T = A^{-1}$. Find $\det(A)$ where A is orthogonal.

$I = AA^{-1} \Rightarrow I = AA^T$

$\Rightarrow |I| = |AA^T| \Rightarrow 1 = |A|\cdot|A^T|$

(but $|A^T| = |A|$)

$\Rightarrow 1 = |A|\cdot|A| \Rightarrow |A|^2 = 1 \Rightarrow |A| = \pm 1$

3