

§(3.1)

#3)  $P(0) = 500, P(10) = 575$   
 $P = 500e^{kt} \Rightarrow k = \frac{1}{10} \ln 1.15$   
 $P(30) = 500e^{3 \ln 1.15} \approx 760 \text{ years}$

#6)  $dS/dt = rS \Rightarrow S = S_0 e^{rt}$   
 (a)  $S_0 = 5000, r = 5.75\% \Rightarrow S(5) = 6665.4$   
 (b)  $S(t) = 10,000 \Rightarrow t = 12 \text{ years}$   
 (c)  $S \approx 6651.82$

#13)  $\frac{dT}{dt} = k(T-10) \Rightarrow T = 10 + ce^{kt}$   
 $T(0) = 70, T(\frac{1}{2}) = 50 \Rightarrow$   
 $c = 60, k = 2 \ln \frac{2}{3} \Rightarrow$   
 $T = 10 + 60 e^{\ln(\frac{2}{3})^{2t}} = 10 + 60 (\frac{2}{3})^{2t}$   
 $T(1) = 10 + 60 (\frac{2}{3})^2 = 36.67$   
 $T(t) = 15 \Rightarrow 15 = 10 + 60 (\frac{2}{3})^{2t}$   
 $\Rightarrow t \approx 3 \text{ minutes}$

#14)  $\frac{dT}{dt} = k(T-5) \Rightarrow T = 5 + ce^{kt}$   
 $T(1) = 55, T(5) = 30 \Rightarrow$   
 $k = -\frac{1}{4} \ln 2, c = 59.4611$   
 $T = 5 + (59.4611) e^{-\frac{1}{4} \ln 2 t}$   
 $\Rightarrow T(0) = 5 + 59.4611 = 64.4611$

#15)  $\frac{dT}{dt} = k(T-100) \Rightarrow T = 100 + ce^{kt}$   
 $T(0) = 20, T(1) = 22 \Rightarrow c = -80$   
 $k = \ln \frac{39}{41}$   
 $T = 90 \Rightarrow t = 82 \text{ seconds}$   
 $T = 98 \Rightarrow t = 145.7 \text{ seconds}$

#20)  $\frac{dA}{dt} = 0 - \frac{A}{50}$   
 $A = ce^{-t/50}$

$A(0) = 30 \Rightarrow c = 30$   
 $A(t) = 30 e^{-t/50}$

#21)  $\frac{dA}{dt} = 10 - \frac{A}{100}$   
 $A = 1000 + ce^{-t/100}$

$A(0) = 0 \Rightarrow c = -1000$   
 $A(t) = 1000 - 1000 e^{-t/100}$   
 $A(5) = 1000 - 1000 e^{-5/100} \approx 48.7$

#23)  $R_{in} = \left(2 \frac{\text{pound}}{\text{gal}}\right) \left(5 \frac{\text{gal}}{\text{min}}\right)$   
 $= 10 \text{ pound/min}$

Amount of mixed solution at time  $t$   
 $is = 500 - (10-5)t$

Concentration at time  $t$   
 $is = \frac{A(t)}{500 - 5t}$

$R_{out} = \left(\frac{A}{500 - 5t}\right) \left(10 \frac{\text{gal}}{\text{min}}\right)$   
 $= \frac{2A}{100 - t}$

$\frac{dA}{dt} = 10 - \frac{2A}{100 - t}$   
 $A' + \left(\frac{2}{100-t}\right)A = 10 \text{ linear}$

$k(t) = (t-100)^{-2} \Rightarrow$   
 $A = 1000 - 10t + c(t-100)^2$

$A(0) = 0 \Rightarrow c = -10$   
 $A = 1000 - 10t - \frac{1}{10}(t-100)^2$   
 $A = 0 \Rightarrow t = 100$