Appendix II

#47) $A = \begin{bmatrix} -1 & 2 \\ -7 & 8 \end{bmatrix}$, $P(\lambda) = |A - \lambda I| = \begin{vmatrix} -1-\lambda & 2 \\ -7 & 8-\lambda \end{vmatrix}$

$= (\lambda+1)(\lambda-8) + 14 = \lambda^2 - 7\lambda + 6$

$= (\lambda - 6)(\lambda - 1)$

$\Rightarrow \lambda = 1, \ \lambda = 6$ are eigenvalues

$\underline{\lambda = 1}$: $\left[\begin{array}{cc|c} -1-1 & 2 & 0 \\ -7 & 8-1 & 0 \end{array}\right] = \left[\begin{array}{cc|c} -2 & 2 & 0 \\ -7 & 7 & 0 \end{array}\right] \xrightarrow{-\frac{1}{2}R_1} \left[\begin{array}{cc|c} 1 & -1 & 0 \\ -7 & 7 & 0 \end{array}\right]$

$\xrightarrow{7R_1 + R_2} \left[\begin{array}{cc|c} 1 & -1 & 0 \\ 0 & 0 & 0 \end{array}\right] \Rightarrow k_1 - k_2 = 0 \Rightarrow k_1 = k_2$

$\overline{K}_1 = \begin{bmatrix} k_1 \\ k_2 \end{bmatrix} = \begin{bmatrix} k_2 \\ k_2 \end{bmatrix} = k_2 \begin{bmatrix} 1 \\ 1 \end{bmatrix} \Rightarrow K_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ eigenvector

$\underline{\lambda = 6}$: $\left[\begin{array}{cc|c} -1-6 & 2 & 0 \\ -7 & 8-6 & 0 \end{array}\right] = \left[\begin{array}{cc|c} -7 & 2 & 0 \\ -7 & 2 & 0 \end{array}\right] \xrightarrow{-R_1 + R_2} \left[\begin{array}{cc|c} -7 & 2 & 0 \\ 0 & 0 & 0 \end{array}\right]$

$\Rightarrow -7k_1 + 2k_2 = 0$ choose $k_1 = 2, \ k_2 = 7$

$K_2 = \begin{bmatrix} 2 \\ 7 \end{bmatrix}$

#55) $A = \begin{bmatrix} -1 & 2 \\ -5 & 1 \end{bmatrix}$, $P(\lambda) = \begin{vmatrix} -1-\lambda & 2 \\ -5 & 1-\lambda \end{vmatrix}$

$= (\lambda+1)(\lambda-1) + 10 = \lambda^2 + 9$

Now, $\lambda = 3i, -3i$ eigenvalues.

$\underline{\text{Case } \lambda = 3i}$: $\left[\begin{array}{cc|c} -1-3i & 2 & 0 \\ -5 & 1-3i & 0 \end{array}\right] \xrightarrow{R_1 \Leftrightarrow R_2} \left[\begin{array}{cc|c} -5 & 1-3i & 0 \\ -1-3i & 2 & 0 \end{array}\right]$

$\xrightarrow{-\frac{1}{5}R_1} \left[\begin{array}{cc|c} 1 & -\frac{1}{5} + \frac{3}{5}i & 0 \\ -1-3i & 2 & 0 \end{array}\right] \xrightarrow{(1+3i)R_1 + R_2} \left[\begin{array}{cc|c} 1 & -\frac{1}{5} + \frac{3}{5}i & 0 \\ 0 & 0 & 0 \end{array}\right]$

$\Rightarrow k_1 + (-\frac{1}{5} + \frac{3}{5}i)k_2 = 0$ Choose $k_2 = 1 \Rightarrow k_1 = \frac{1}{5} - \frac{3}{5}i$

$K = \begin{bmatrix} \frac{1}{5} - \frac{3}{5}i \\ 1 \end{bmatrix}$ is an eigenvector for $\lambda = 3i$