8) Write in $L(y) = g(x)$ and factor L.

$y''' + 4y'' + 3y' = x^2\cos x - 3x$

using $y' = Dy$, $y'' = D^2 y$
$y''' = D^3 y$

D.E $\Rightarrow$

$(D^3 + 4D^2 + 3D)\,y = x^2\cos x - 3x$

or $D(D^2 + 4D + 3D)\; y = x^2\cos x - 3x$

or $\boxed{D(D+1)(D+3)\,y = x^2\cos x - 3x}$

Additional Note: Annihilator for R.H.S is $D^2(D^2+1)^3$

13) Verify that $y = e^{2x} + 3e^{-5x}$ has annihilator is $(D-2)(D+5)$

$(D-2)(D+5)(e^{2x} + 3e^{-5x})$

$= (D-2)\left[(D+5)e^{2x} + 3(D+5)e^{-5x}\right]$

$= (D-2)\left[2e^{2x} + 5e^{2x} + 3(-5e^{-5x} + \cancel{5e^{-5x}})\right]$

$= 2\cdot 2e^{2x} - 4e^{2x} + 5(2e^{2x} - 2e^{2x})$

$= 0$. Hence the result.

22) Find annihilator.

$8x - \sin x + 10\cos 5x$

annihilator for $8x$: D^2

annihilator for $-\sin x$: (D^2+1)

annihilator for $10\cos 5x$: (D^2+25)

Thus required annihilator is

$D^2(D^2+1)(D^2+25)$.

24) Find annihilator : $(2 - e^x)^2$

$= 4 - 2e^x + e^{2x}$

The annihilator is therefore

$D(D-1)(D-2)$.

41) $y''' + y'' = 8x^2$ —(1)

Step 1 Homogeneous D.E

$m^3 + m^2 = 0 \Rightarrow m^2(m+1) = 0$

$\Rightarrow m = 0, 0, -1$

$y_c = C_1 + C_2 x + C_3 e^{-x}$

Step 2: Annihilator of R.H.S is D^3

Apply to D.E

$D^3(D^3 + D^2)y = D^3(8x^2) = 0$

Auxiliary eqn: $m^3(m^3 + m^2) = 0$

$m = 0, 0, 0, 0, 0, -1$

$y = \underbrace{A + Bx}_{y_c} + Cx^2 + Dx^3 + Ex^4 + \underbrace{Fe^{-x}}_{y_c}$

Cancel out terms common with y_c

$y_p = Cx^2 + Dx^3 + Ex^4$

$y_p' = 2Cx + 3Dx^2 + 4Ex^3$

$y_p'' = 2C + 6Dx + 12Ex^2$

$y_p''' = 6D + 24Ex$

D.E (1) $\Rightarrow$

$6D + 24Ex + 2C + 6Dx + 12Ex^2 = 8x^2$

Compare coefficients of like terms

1 : $6D + 2C = 0$ —(a)

x : $6D + 24E = 0$ —(b) $\Rightarrow 6D = -24E$

x^2 : $12E = 8$ —(c)

$\Rightarrow E = \frac{8}{12} \Rightarrow \boxed{E = \frac{2}{3}}$

From (b) $\Rightarrow D = -\frac{24}{6}E = -\frac{\cancel{24}^{4}}{\cancel{6}}\cdot\frac{2}{3} = \boxed{\frac{-8}{3}}$

From (a) $\Rightarrow \boxed{C = 8}$

Hence $\boxed{y_p = \frac{2}{3}x^4 - \frac{8}{3}x^3 + 8x^2}$

Step 3 $y = y_c + y_p$ so

$y = C_1 + C_2 x + C_3 e^{-x} + \frac{2}{3}x^4 - \frac{8}{3}x^3 + 8x^2$.

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(4.5 contd)

48) $y''+4y = 4\cos x + 3\sin x - 8$

Step 1 Homogeneous D.E : $y''+4y=0$

Auxiliary Eqn : $m^2+4=0$

$\Rightarrow m = \pm 2i$

$y_c = C_1\cos 2x + C_2\sin 2x$

Step 2 : Annihilator : $D(D^2+1)$

Apply on D.E

$D(D^2+1)(D^2+4)y = D(D^2+1)[4\cos x + 3\sin x - 8]$

$= 0$

Auxiliary eqn is

$m(m^2+1)(m^2+4) = 0$

$m = 0, \pm i, \pm 2i$

$y = A + B\cos x + C\sin x + \underbrace{D\cos 2x + E\sin 2x}_{y_c}$

Cancel terms contained in y_c:

Thus $y_p = A + B\cos x + C\sin x$

$y_p' = -B\sin x + C\cos x$

$y_p'' = -B\cos x - C\sin x$

D.E $\Rightarrow -B\cos x - C\sin x + 4A + 4B\cos x + 4C\sin x = 4\cos x + 3\sin x - 8$

Compare coefficients of like terms:

1 : $4A = -8 \Rightarrow A = -2$

$\cos x$: $-B + 4B = 4 \Rightarrow B = \frac{4}{3}$

$\sin x$: $-C + 4C = 3 \Rightarrow C = 1$

Thus $y_p = -2 + \frac{4}{3}\cos x + \sin x$

Step 3 : $y = y_c + y_p$

$y = C_1\cos 2x + C_2\sin 2x - 2 + \frac{4}{3}\cos x + \sin x$

64) $y^{(4)} - 4y'' = 5x^2 - e^{2x}$

Step 1 . Homogeneous DE : $y^{(4)} - 4y'' = 0$

Auxiliary Eqn : $m^4 - 4m^2 = 0$

$m = 0, 0, -2, 2$

$y_c = C_1 + C_2 x + C_3 e^{-2x} + C_4 e^{2x}$

Step 2 : Annihilator of R.H.S

$D^3(D-2)$

Apply on D.E

$D^3(D-2)[D^4 - 4D^2]y = D^3(D-2)[5x^2 - e^{2x}]$

$= 0$

Auxiliary equation is

$m^3(m-2)(m^4 - 4m^2) = 0$

$m = \underbrace{0, 0, 0, 0, 0}_{5 \text{ values}}, \quad m = 2, 2, -2$

$y = \underbrace{A + Bx}_{y_c} + Cx^2 + Dx^3 + Ex^4 + \underbrace{Fe^{2x}}_{y_c} + Gxe^{2x} + He^{-2x}$

Cancel terms common with y_c

$y_p = Cx^2 + Dx^3 + Ex^4 + Gxe^{2x}$

$y_p' = 2Cx + 3Dx^2 + 4Ex^3 + Ge^{2x} + 2Gxe^{2x}$

$y_p'' = 2C + 6Dx + 12Ex^2 + 2Ge^{2x} + 2Ge^{2x} + 4Gxe^{2x}$

$= 2C + 6Dx + 12Ex^2 + 4Ge^{2x} + 4Gxe^{2x}$

$y_p''' = 6D + 24Ex + 8Ge^{2x} + 4Ge^{2x} + 8Gxe^{2x}$

$= 6D + 24Ex + 12Ge^{2x} + 8Gxe^{2x}$

$y_p^{(4)} = 24E + 24Ge^{2x} + 8Ge^{2x} + 16Gxe^{2x}$

$= 24E + 32Ge^{2x} + 16Gxe^{2x}$

Put in D E

$24E + 32Ge^{2x} + 16Gxe^{2x} - 8C - 24Dx - 48Ex^2 - 16Ge^{2x} - 16Gxe^{2x} = 5x^2 - e^{2x}$

Compare Coefficients

1 : $24E - 8C = 0 \Rightarrow C = 3E$

x : $-24D = 0 \Rightarrow D = 0$

x^2 : $-48E = 5 \Rightarrow E = -\frac{5}{48}$

e^{2x} : $32G - 16G = -1 \Rightarrow G = -\frac{1}{16}$

xe^{2x} : $16G - 16G = 0$

$C = 3E = -\frac{5}{16}$.

$y_p = -\frac{5}{16}x^2 - \frac{5}{48}x^4 - \frac{1}{16}xe^{2x}$

Step 3 $y = y_c + y_p$

$= C_1 + C_2 x + C_3 e^{-2x} + C_4 e^{2x} - \frac{5}{16}x^2 - \frac{5}{48}x^4 - \frac{1}{16}xe^{2x}$.