Math 202

4.3

4) $y'' - 3y' + 2y = 0$

The auxiliary equation is

$m^2 - 3m + 2 = 0$

$$m = \frac{3 \pm \sqrt{9-8}}{2} = \frac{3 \pm 1}{2} = 2, 1$$

Thus $y = C_1 e^{x} + C_2 e^{2x}$.

9) $y'' + 49y = 0$

The auxiliary equation is

$m^2 + 49 = 0$

$\Rightarrow m = \pm 7i = \alpha \pm i\beta$

$\alpha = 0, \ \beta = 7$

Thus $y_1 = e^{0x} \cos 7x$, $y_2 = e^{0x} \sin 7x$

$y = C_1 \cos 7x + C_2 \sin 7x$

12) $2y'' + 2y' + y = 0.$

The auxiliary equation is

$2m^2 + 2m + 1 = 0$

$$m = \frac{-2 \pm \sqrt{4-8}}{4}$$

$$= \frac{-2 \pm 2i}{4} = \frac{-1}{2} \pm \frac{i}{2} = \alpha \pm i\beta$$

$\alpha = -\frac{1}{2}, \quad \beta = \frac{1}{2}$

Thus

$y = C_1 e^{-x/2} \cos \frac{x}{2} + C_2 e^{-x/2} \sin x/2$

$= e^{-x/2}(C_1 \cos \frac{x}{2} + C_2 \sin \frac{x}{2})$.

15) $y'' + 4y'' - 5y' = 0$

The auxiliary equation is

$m^3 + 4m^2 - 5m = 0$ or

$m(m^2 + 4m - 5) = 0$

$$m = 0, \quad m = \frac{-4 \pm \sqrt{16+20}}{2}$$

$$= \frac{-4 \pm 6}{2} = -5, +1$$

$m = 0, +1, -5$

Thus $y = C_1 + C_2 e^{+x} + C_3 e^{-5x}$.

20) $\frac{d^3x}{dt^3} - \frac{d^2x}{dt^2} - 4x = 0$

Auxiliary eqn: $m^3 - m^2 - 4 = 0$

By inspection $m = 2$ is a root.

By long division $m^3 - m^2 - 4 = (m-2)(m^2+m+2)$

Thus $m = 2$, $m^2 + m + 2 = 0$ or

$$m = \frac{-1 \pm \sqrt{1-8}}{2} = \frac{-1 \pm \sqrt{7}i}{2}$$

$y = C_1 e^{2x} + C_2 e^{-\frac{1}{2}x} \cos \frac{\sqrt{7}}{2}x + C_3 e^{-\frac{1}{2}x} \sin \frac{\sqrt{7}}{2}x.$

34) $y'' - 2y' + y = 0, \quad y(0) = 5$

$y'(0) = 10.$

Auxiliary eqn

$m^2 - 2m + 1 = 0 \Rightarrow m = \frac{2 \pm \sqrt{4-4}}{2}$

$= 1, 1$

$y = C_1 e^x + C_2 x e^x$

$y(0) = 5 \Rightarrow C_1 + 0 = 5 \Rightarrow C_1 = 5$

$y' = 5 e^x + C_2 e^x + C_2 x e^x$

$y'(0) = 10 \Rightarrow 5 + C_2 = 10 \Rightarrow C_2 = 5$

$\Rightarrow y = 5e^x + 5x e^x.$

Math 20

4.3 (contd)

40) $y'' - 2y' + 2y = 0$, $y(0) = 1$, $y(\pi) = 1$.

Auxiliary equation

$m^2 - 2m + 2 = 0$

$m = \frac{2 \pm \sqrt{4-8}}{2} = \frac{2 \pm 2i}{2}$

$= 1 \pm i = \alpha \pm i\beta$

$y = C_1 e^x \cos x + C_2 e^x \sin x$

$y(0) = 1 \Rightarrow C_1 = 1$

$y(\pi) = 1 \Rightarrow (1) e^{\pi}(-1) + C_2 e^{\pi}(0) = 1$

C_2 can not be determined.

(Possible misprint.)

50) The roots of a cubic auxiliary equation with real coefficients: $m_1 = -\frac{1}{2}$, $m_2 = 3+i$

As third root should be $m_3 = 3 - i$ we have auxiliary eqn as

$(m + \frac{1}{2})(m - 3 - i)(m - 3 + i) = 0$

$\Rightarrow (m + \frac{1}{2})((m-3)^2 + 1) = 0$

$\Rightarrow (m + \frac{1}{2})(m^2 - 6m + 10) = 0$

or $(m^3 - 6m^2 + 10m + \frac{1}{2}m^2 - 3m + 5) = 0$

or $m^3 - \frac{11}{2}m^2 + 7m + 5 = 0$

This corresponds to

$\frac{d^3y}{dx^3} - \frac{11}{2}\frac{d^2y}{dx^2} + 7\frac{dy}{dx} + 5y = 0$

or $2y''' - 11y'' + 14y' + 10y = 0$.

Any multiple of this equation will also have same solutions

49) $m_1 = 4$, $m_2 = m_3 = -5$

Auxiliary eqn is

$(m-4)(m+5)^2 = 0$

or $(m-4)(m^2 + 10m + 25) = 0$

or $m^3 + 10m^2 + 25m - 4m^2 - 40m - 100 = 0$

$\Rightarrow m^3 + 6m^2 - 15m - 100 = 0$

D.E is

$\frac{d^3y}{dx^3} + 6\frac{d^2y}{dx^2} - 15\frac{dy}{dx} - 100y = 0$

51) $y''' + 6y'' + y' - 34y = 0$.

One solution $y_1 = e^{-4x}\cos x$

Thus $m = -4 \pm i$ are complex conjugate roots. These correspond to

$(m + 4 - i)(m + 4 - i) = (m+4)^2 + 1$

$= m^2 + 8m + 17$

Thus $\frac{m^3 + 6m^2 + m - 34}{m^2 + 8m + 17}$ gives third factor which is (by division)

$(m - 2)$ i.e. $m = 2$.

Hence general solution for $m = 2, -4 \pm i$ is

$y = C_1 e^{2x} + C_2 e^{-4x}\cos x + C_3 e^{-4x}\sin x$.

Main Idea If $m = m_1, m_2$ are roots then $(m - m_1)(m - m_2)$ are factors.