#11) $x' = \begin{bmatrix} -3 & 0 & -4 \\ -1 & -1 & -1 \\ 1 & 0 & 1 \end{bmatrix} x$

To find eigenvalues:

$$|A-\lambda I| = \begin{vmatrix} -3-\lambda & 0 & -4 \\ -1 & -1-\lambda & -1 \\ 1 & 0 & 1-\lambda \end{vmatrix}$$

$\boxed{\text{expand along first row}}$

$$= (-3-\lambda)\begin{vmatrix} -1-\lambda & -1 \\ 0 & 1-\lambda \end{vmatrix} - 0 \times \begin{vmatrix} \end{vmatrix} - 4\begin{vmatrix} -1 & -1-\lambda \\ 1 & 0 \end{vmatrix}$$

$$= (-3-\lambda)\left[(-1-\lambda)(1-\lambda)\right] - 4\left[-(-1-\lambda)\right]$$

$\boxed{\text{DO NOT OPEN brackets}}$ (Note)

$$= (-3-\lambda)\left[(\lambda+1)(\lambda-1)\right] - 4\left[\lambda+1\right]$$

$$= (\lambda+1)\left[(-3-\lambda)(\lambda-1) - 4\right]$$

$$= (\lambda+1)\left[-\lambda^2 - 2\lambda - 1\right]$$

$$= -(\lambda+1)(\lambda^2+2\lambda+1) = -(\lambda+1)^3$$

$\lambda = -1, -1, -1$

#7) Find eigenvalues $A = \begin{bmatrix} 4 & -2 & 1 \\ 2 & 0 & 1 \\ 2 & -2 & 3 \end{bmatrix}$

$$|A - \lambda I| = \begin{vmatrix} 4-\lambda & -2 & 1 \\ 2 & -\lambda & 1 \\ 2 & -2 & 3-\lambda \end{vmatrix}$$

$\boxed{\text{expand along first row}}$

$$= (4-\lambda)\begin{vmatrix} -\lambda & 1 \\ -2 & 3-\lambda \end{vmatrix} - (-2)\begin{vmatrix} 2 & 1 \\ 2 & 3-\lambda \end{vmatrix} + (1)\begin{vmatrix} 2 & -\lambda \\ 2 & -2 \end{vmatrix}$$

$$= (4-\lambda)\left[-\lambda(3-\lambda)+2\right] + 2\left[6-2\lambda-2\right] + \left[-4+2\lambda\right]$$

$$= (4-\lambda)\left[\lambda^2-3\lambda+2\right] + 2\left[4-2\lambda\right] + \left[-4+2\lambda\right]$$

$\boxed{\text{DO NOT OPEN BRACKETS}}$ import

$$= (4-\lambda)\left[(\lambda-2)(\lambda-1)\right] + 4\left[2-\lambda\right] + 2\left[\lambda-2\right]$$

$$= (\lambda-2)\left[(4-\lambda)(\lambda-1) - 4 + 2\right]$$

$$= (\lambda-2)\left[-\lambda^2 + 5\lambda - 6\right]$$

$$= -(\lambda-2)(\lambda^2 - 5\lambda + 6)$$

$$= -(\lambda-2)(\lambda-3)(\lambda-2)$$

$$\Longrightarrow \lambda = 2, 2, 3$$