Name: KEY	**ID:** KEY	**serial:**

MATH-202 **Term-072** **QUIZ-3** **Form(B)**

Q1:True of False

$$W = \begin{vmatrix} x^2 - x & 2x^2 - 3x \\ 2x - 1 & 4x - 3 \end{vmatrix} = (x^2 - x)(4x-3) - (2x-1)(2x^2-3x)$$
$$= (4x^3 - 3x^2 - 4x^2 + 3x) - (4x^3 - 6x^2 - 2x^2 + 3x) = (-7x^2) - (-8x^2) = x^2 \neq 0$$

1) The functions $f_1(x) = x^2 - x, f_2(x) = 2x^2 - 3x$ are linearly independent (T)

2) The functions $f_1(x) = e^{3x}, f_2(x) = e^{-3x}$ from a fundamental set of solutions for the DE $y''' - 9y' = 0$ (F)

3) If $W(y_1, y_2, y_3) = x$ then y_1, y_2, y_3 are linearly dependent on the interval I=(-2,2). (F)

Q2) Solve $y''' - 4y' = 0$

$$(D^3 - 4D)y = 0$$

Aux eq: $m^3 - 4m = 0$

$m(m-2)(m+2) = 0$

roots : $0, \; 2, \; -2$

Sol : $1, \; e^{2x}, \; e^{-2x}$

The genral sol is

$$y = c_1 + c_2 e^{2x} + c_3 e^{-2x}$$

Q2) Solve $xy'' - y' = 0$ given that $y_1 = x^2$ is a particular solution.

divid by x

$$y'' - \frac{1}{x} y' = 0 \quad \Rightarrow \quad P(x) = -\frac{1}{x}$$

$$K(x) = e^{-\int P(x)\,dx} = e^{\int \frac{dx}{x}} = x$$

By formula (5) in sec 4.2

$$y_2 = y_1(x) \int \frac{K(x)}{y_1^2(x)}\,dx = x^2 \int \frac{x}{(x^2)^2}\,dx$$

$$= x^2 \int \frac{dx}{x^3} = x^2 \int x^{-3}\,dx = x^2 \left(\frac{x^{-2}}{-2} \right) = -\frac{1}{2}$$

Hence, $y_1 = x^2$, $y_2 = -\frac{1}{2}$

The general sol for $xy'' - y' = 0$

$$y = c_1 x^2 - \frac{1}{2} \tilde{C}_2$$

or

$$\boxed{y = c_1 x^2 + c_2}$$