Name: KEY	ID: KEY	serial:

MATH-202 Term-072 QUIZ-5 Form(B)

1) Solve $X' = AX$

$$A = \begin{bmatrix} 6 & -1 \\ 5 & 2 \end{bmatrix}$$

Hint: $\begin{bmatrix} 2+i \\ 5 \end{bmatrix}$ is an eigenvector for the matrix A associated with the eigenvalue $\lambda = 4 + i$

$\lambda = 4 + i \Rightarrow \alpha = 4, \quad \beta = 1$

$K = \begin{bmatrix} 2+i \\ 5 \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} i \Rightarrow B_1 = \begin{bmatrix} 2 \\ 5 \end{bmatrix}, \; B_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

using Theorem 8.9/p.348

$X_1 = [B_1 \cos\beta t - B_2 \sin\beta t] e^{\alpha t}$

$X_2 = [B_2 \cos\beta t + B_1 \sin\beta t] e^{\alpha t}$

So, $X_1 = \left(\begin{bmatrix} 2 \\ 5 \end{bmatrix} \cos t - \begin{bmatrix} 1 \\ 0 \end{bmatrix} \sin t \right) e^{4t}$

$= \begin{bmatrix} 2\cos t - \sin t \\ 5\cos t \end{bmatrix} e^{4t}$

$X_2 = \left(\begin{bmatrix} 1 \\ 0 \end{bmatrix} \cos t + \begin{bmatrix} 2 \\ 5 \end{bmatrix} \sin t \right) e^{4t}$

$= \begin{bmatrix} \cos t + 2\sin t \\ 5\sin t \end{bmatrix} e^{4t}$

Now, The general sol $X = c_1 X_1 + c_2 X_2$

2) Solve $X' = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix} X$

Eigenvalues. $P(\lambda) = |A - \lambda I| = \begin{vmatrix} 1-\lambda & -1 \\ 0 & 1-\lambda \end{vmatrix} = (1-\lambda)^2$

$\Rightarrow \lambda = 1, 1$ repeated

eigenvectors $[A - \lambda I \mid 0] = [A - I \mid 0] = \left[\begin{array}{cc|c} 1-1 & -1 & 0 \\ 0 & 1-1 & 0 \end{array}\right]$

$= \left[\begin{array}{cc|c} 0 & -1 & 0 \\ 0 & 0 & 0 \end{array}\right] \xrightarrow{-R_1} \left[\begin{array}{cc|c} 0 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right] \Rightarrow k_2 = 0$

$\begin{bmatrix} k_1 \\ k_2 \end{bmatrix} = \begin{bmatrix} k_1 \\ 0 \end{bmatrix} = k_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} \Rightarrow K = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ is an eigenvector

Now, $K = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ is an eigenvector associated with $\lambda = 1$

$\Rightarrow X_1 = K e^{\lambda t} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} e^t = \begin{bmatrix} e^t \\ 0 \end{bmatrix}$

To find the second solution we need to find the vector P.

$[A - \lambda I \mid K] = \left[\begin{array}{cc|c} 0 & -1 & 1 \\ 0 & 0 & 0 \end{array}\right] \xrightarrow{-R_1} \left[\begin{array}{cc|c} 0 & 1 & -1 \\ 0 & 0 & 0 \end{array}\right] \Rightarrow p_2 = -1$

$\begin{bmatrix} p_1 \\ p_2 \end{bmatrix} = \begin{bmatrix} p_1 \\ -1 \end{bmatrix}$ let us choose $p_1 = 0 \Rightarrow P = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$

$X_2 = K t e^{\lambda t} + P e^{\lambda t} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} t e^t + \begin{bmatrix} 0 \\ -1 \end{bmatrix} e^t$

$= \begin{bmatrix} t e^t \\ -e^t \end{bmatrix}$

The general sol is $X = c_1 X_1 + c_2 X_2$

$= \begin{bmatrix} c_1 e^t + c_2 t e^t \\ -c_2 e^t \end{bmatrix}$