Name: KEY	ID: KEY

MATH 202 **Term 072** **QUIZ 4**

1) Find the **recurrence relation** of the given differential equation about the ordinary point $x = 0$. $y'' - xy = 0$ [DONOT SOLVE THE PROBLEM]

$$y = \sum_{n=0}^{\infty} c_n x^n \quad , \quad y'' = \sum_{n=2}^{\infty} n(n-1) c_n x^{n-2}$$

$$LHS = y'' - xy = \sum_{n=2}^{\infty} n(n-1) c_n x^{n-2} - \sum_{n=0}^{\infty} c_n x^{n+1}$$

$$= 2(1) c_2 x^0 + \sum_{n=3}^{\infty} n(n-1) c_n x^{n-2} - \sum_{n=0}^{\infty} c_n x^{n+1}$$

$k = n-2$ $\qquad k = n+1$

$$= 2c_2 + \sum_{k=1}^{\infty} (k+1)(k+2) C_{k+2} x^k - \sum_{k=1}^{\infty} C_{k-1} x^k$$

$$= 2c_2 + \sum_{k=1}^{\infty} \left[(k+1)(k+2) C_{k+2} - C_{k-1}\right] x^k = 0$$

$$\boxed{C_2 = 0 \quad \text{and} \quad C_{k+2} = \frac{1}{(k+1)(k+2)} C_{k-1} \quad k = 1, 2, 3, \ldots}$$

2) Find two power series solutions of the given differential equation about the ordinary point x=0. $y'' + x^2 y = 0$ (Show all your work)

[given that $2c_2 = 0, \quad 6c_3 = 0.$ $(k+2)(k+1)C_{k+2} + C_{k-2} = 0. \quad k = 2, 3, 4, 5, \ldots$ is the recurrence relation]

$c_2 = 0$ and $c_3 = 0$

$$C_{k+2} = \frac{-1}{(k+1)(k+2)} C_{k-2} \qquad k = 2, 3, 4, 5 \ldots$$

$C_4 = \frac{-1}{3 \cdot 4} C_0$	$C_5 = \frac{-1}{4 \cdot 5} C_1$	$C_6 = \frac{-1}{5 \cdot 6} C_2 = 0$	$C_7 = \frac{-1}{6 \cdot 7} C_3 = 0$
$C_8 = \frac{-1}{7 \cdot 8} C_4$	$C_9 = \frac{-1}{8 \cdot 9} C_5$	$C_{10} = \frac{-1}{9 \cdot 10} C_6 = 0$	$C_{11} = \sim C_7 = 0$
$\vdots$	$\vdots$	$= 0$	$= 0$
$C_{4m} = \frac{-1}{(4m-1)(4m)} C_{4m-4}$	$C_{4m+1} = \frac{-1}{(4m)(4m+1)} C_{4m-3}$		

$$C_{4m} = \frac{(-1)^m}{[3 \cdot 7 \cdots (4m-1)][4 \cdot 8 \cdots 4m]} C_0 \qquad m = 1, 2, 3, \ldots$$

$$= \frac{(-1)^m}{[3 \cdot 7 \cdots (4m-1)]\, 4^m \cdot [1 \cdot 2 \cdots m]} C_0$$

$$\boxed{C_{4m} = \frac{(-1)^m}{4^m \cdot m!\, [3 \cdot 7 \cdots (4m-1)]} C_0 \qquad m = 1, 2, 3, \ldots} \quad (1)$$

$$C_{4m+1} = \frac{(-1)^m}{[4 \cdot 8 \cdots (4m)][5 \cdot 9 \cdots (4m+1)]} C_1 \qquad m = 1, 2, 3, \ldots$$

$$\boxed{C_{4m+1} = \frac{(-1)^m}{4^m \cdot m!\, [5 \cdot 9 \cdots (4m+1)]} C_1 \qquad m = 1, 2, 3, \ldots} \quad (2)$$

$$\boxed{C_{4m+2} = 0, \quad C_{4m+3} = 0 \qquad m = 1, 2, 3, \ldots} \quad (3)$$

$$y = \sum_{n=0}^{\infty} c_n x^n = \sum_{m=0}^{\infty} C_{4m} x^{4m} + \sum_{m=0}^{\infty} C_{4m+1} x^{4m+1} + \sum_{m=0}^{\infty} C_{4m+2} x^{4m+2}$$

$$+ \sum_{m=0}^{\infty} C_{4m+3} x^{4m+3}$$

$$= \left(C_0 + \sum_{m=1}^{\infty} C_{4m} x^{4m}\right) + \left(C_1 x + \sum_{m=1}^{\infty} C_{4m+1} x^{4m+1}\right) + 0 + 0$$

$$= C_0 \left[1 + \sum_{m=1}^{\infty} \frac{(-1)^m x^{4m}}{4^m \cdot m!\, [3 \cdot 7 \cdots (4m-1)]}\right] +$$

$$+ C_1 \left[x + \sum_{m=1}^{\infty} \frac{(-1)^m x^{4m+1}}{4^m \cdot m!\, [5 \cdot 9 \cdots (4m+1)]}\right] = C_0 y_1 + C_1 y_2$$