Math 202

2.2

8) $e^x y \frac{dy}{dx} = e^{-y} + e^{-2x-y}$

$= e^{-y} + e^{-y} e^{-2x}$

thus

$e^x y \frac{dy}{dx} = e^{-y}(1+e^{-2x})$

Separating variables

$y e^y dy = \frac{1+e^{-2x}}{e^x} dx$

$= (e^{-x} + e^{-3x}) dx$

Integrate both sides

$\int \underbrace{y}_{u} \underbrace{e^y dy}_{dv} = \int (e^{-x} + e^{-3x}) dx$

$y e^y - \int e^y dy = -e^{-x} - \frac{1}{3} e^{-3x} + C$

or $y e^y - e^y = -e^{-x} - \frac{1}{3} e^{-3x} + C.$

14) $x(1+y^2)^{1/2} dx = y(1+x^2)^{1/2} dy$

separating variables

$\frac{x\, dx}{(1+x^2)^{1/2}} = \frac{y\, dy}{(1+y^2)^{1/2}}$

Integrate

$\frac{1}{2} \int \frac{2x\, dx}{(1+x^2)^{1/2}} = \frac{1}{2} \int \frac{2y\, dy}{(1+y^2)^{1/2}}$

or $2(1+x^2)^{1/2} = 2(1+y^2)^{1/2} + C.$

which can be simplified if desired.

23) $\frac{dx}{dt} = 4(x^2+1)$, $x(\frac{\pi}{4}) = 1.$

This gives $\frac{dx}{x^2+1} = 4\, dt$

Integrate: $\int \frac{dx}{x^2+1} = \int 4\, dt$

$\Rightarrow \tan^{-1} x = 4t + C$

Using $x(\frac{\pi}{4}) = 1$ i.e $t = \frac{\pi}{4}$, $x = 1$

$\tan^{-1} 1 = 4 \cdot \frac{\pi}{4} + C$

or $\frac{\pi}{4} = \pi + C \Rightarrow C = -\frac{3}{4}\pi$

Implicit solution: $\tan^{-1} x - 4t = \frac{3}{4}\pi$

Explicit solution: $x = \tan(4t - \frac{3}{4}\pi).$

27) $\sqrt{1-y^2}\, dx - \sqrt{1-x^2}\, dy = 0$, $y(0) = \frac{\sqrt{3}}{2}$

Separate variables

$\frac{dx}{\sqrt{1-x^2}} = \frac{dy}{\sqrt{1-y^2}}$

Integrate:

$\int \frac{dx}{\sqrt{1-x^2}} = \int \frac{dy}{\sqrt{1-y^2}}$

$\Rightarrow \sin^{-1} x + A = \sin^{-1} y$

Recall: $\cos(\sin^{-1} x) = \sqrt{1-x^2}$

$x = 0$, $y = \frac{\sqrt{3}}{2}$ gives

$\sin^{-1}(\frac{\sqrt{3}}{2}) = A \Rightarrow A = \pi/3$

$\sin^{-1} y = \sin^{-1} x + \frac{\pi}{3}$

$y = \sin(\sin^{-1} x + \frac{\pi}{3})$

$= \sin(\sin^{-1} x) \cos \pi/3 + \cos(\sin^{-1} x) \sin \pi/3$

$= \frac{1}{2} x + \frac{\sqrt{3}}{2} \sqrt{1-x^2}$