King Fahd University of Petroleum and Minerals
Department of Mathematical Sciences
Math 202 Final Exam
Semester II, 2008- (072)
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Name:	
ID:	

Q		Points
1		**20**
2		**20**
3		**32**
4		**32**
5		**24**
6		**10**
7		**10**
8		**15**
9		**10**
10		**10**
11		**15**
12		**10**
13		**10**
14		**10**
15		**22**
Bonus		10
Total		**250**

(Q1) Find the general solution of the system: $X' = \begin{pmatrix} 2 & 0 \\ 1 & 2 \end{pmatrix} X$

show all your work

$X =$

(Q2) Find the general solution of the system: $X' = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} X$

show all your work

(Q3) Use Variation of Parameters to solve the system:

$$X' = \begin{pmatrix} 1 & -2 \\ 1 & -1 \end{pmatrix} X + \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

(show all your work)

Hint: $\begin{bmatrix} 1+i \\ 1 \end{bmatrix}$ is an eigenvector for the matrix A associated with the eigenvalue $\lambda = i$

Show all your work

(Q4) $x = 0$ is a regular singular point of the DE

$$xy'' - 2y = 0$$

Use the method of Frobenius to obtain two linearly independent series solutions about $x = 0$. [Note: Find the first five coefficients of each series]

(Q5) Consider the following DE: $y'' + (\sin x)y = 0$

a) Find two power series solutions of the given DE about the ordinary point $x = 0$

b) Find interval of convergence of power series solutions.

[**Hint**: $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots\ldots$]

(show all your work) [Note: Find the first four coefficients of each series]

(Q6) $x_0 = 0$ is a regular singular point of the DE

$$4x^2 y'' + 4xy' + (4x^2 - 1)y = 0$$

Find the indicial roots of the singularity.

(a) $r_1 = 1$, $r_2 = -1$

(b) $r_1 = \frac{1}{2}$, $r_2 = \frac{-1}{2}$

(c) $r_1 = \frac{1}{4}$, $r_2 = \frac{-1}{4}$

(d) $r_1 = 0$, $r_2 = \frac{-1}{2}$

(Q7)The form of two linearly independent solutions of the DE

$$xy'' + (1 - x)y' - y = 0$$

(r_1, r_2 are the indicial roots)

(a) $y_1 = \sum_{n=0}^{\infty} c_n x^{n+r_1}, c_0 \neq 0, y_2 = \sum_{n=0}^{\infty} b_n x^{n+r_2}, b_0 \neq 0$

(b) $y_1 = \sum_{n=0}^{\infty} c_n x^{n+r_1}, c_0 \neq 0, y_2 = Cy_1(x)\ln x + \sum_{n=0}^{\infty} b_n x^{n+r_1}, b_0 \neq 0$

(c) $y_1 = \sum_{n=0}^{\infty} c_n x^{n+r_1}, c_0 \neq 0, y_2 = y_1(x)\ln x + \sum_{n=1}^{\infty} b_n x^{n+r_1}$

(d) none of the above

(Q8) Without solving classify each of the following equations as to: (circle the appropriate word)

(a) separable (b) Exact (c) Bernolli

(I)	$(x+y)^2dx+(2xy+x^2-1)dy=0$	separable	exact	Bernolli
(II)	$y'+\frac{y}{x}=xy^2$	separable	exact	Bernolli
(III)	$y'=\frac{y+1}{x}$	separable	exact	Bernolli
(IV)	$y'=(x+1)^2$	separable	exact	Bernolli

(Q9) Transform the system

$$3y_1'' - 5y_2' + t^2 = 0$$

$$7y_2'' + 5y_1' = e^t$$

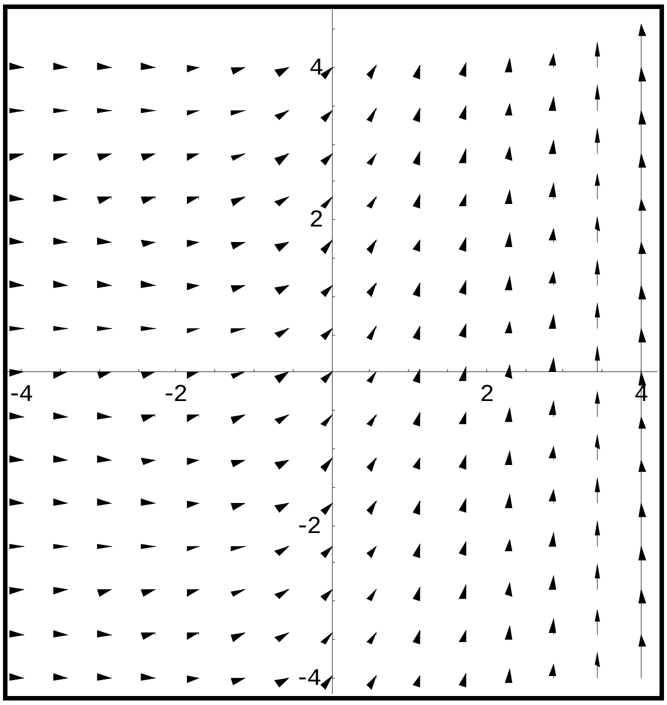
Into a first-order system

$$X'(t) = AX(t) + F(t).$$

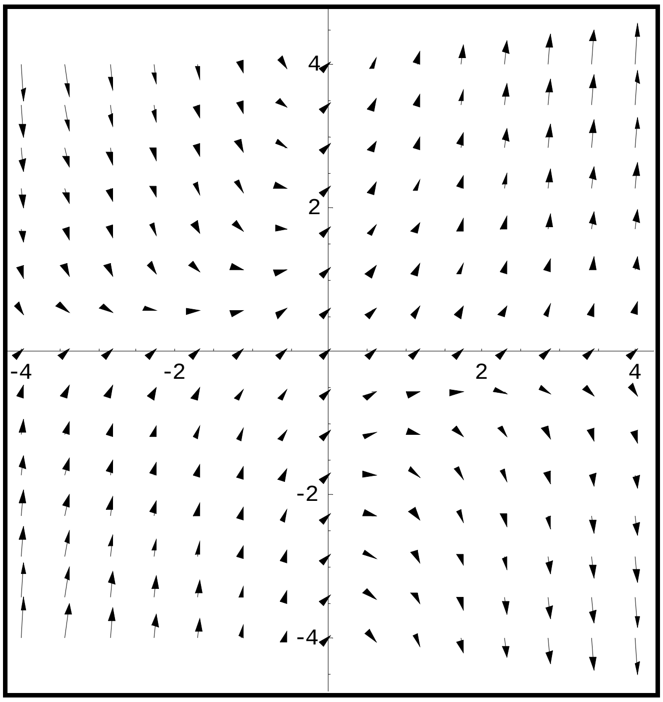
$A =$	$F(t) =$

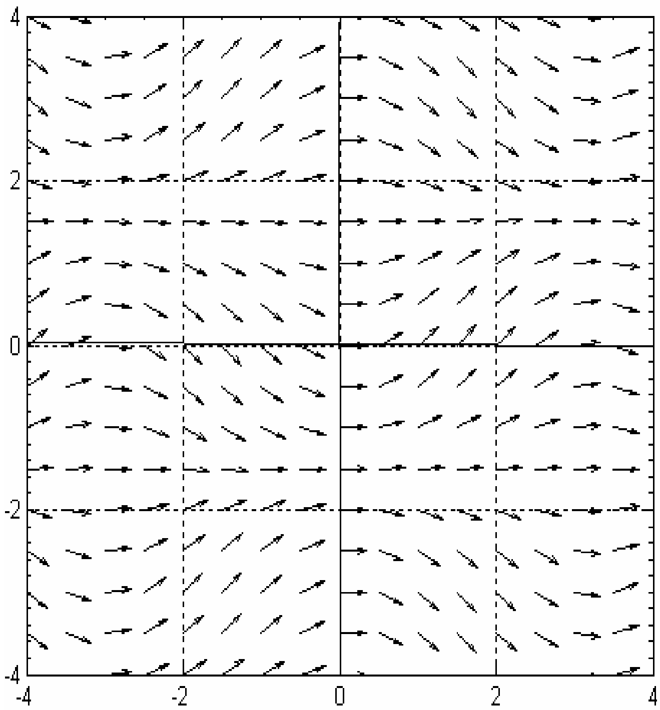
(Q10) The computer-generated directional field of $y' = x^2 y^2$ is :

(a)

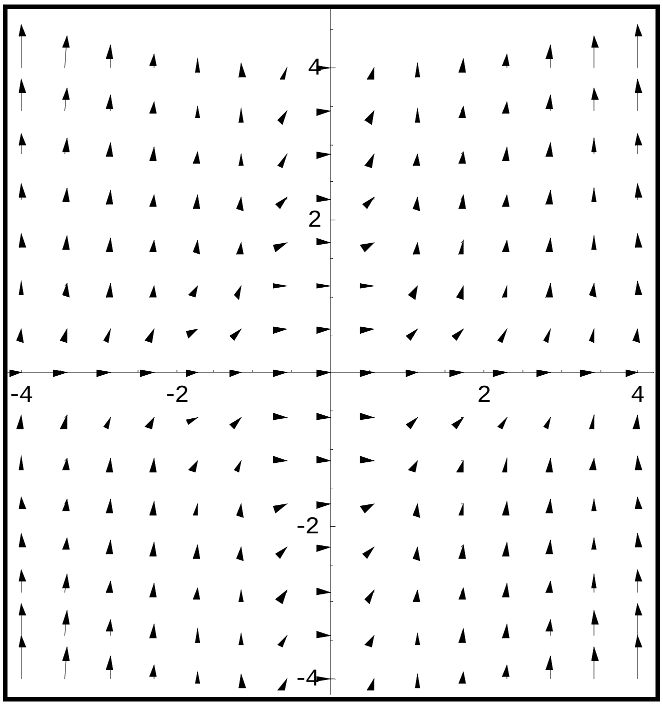


(b)





(c)



(d)

(Q11) State the order of the given differential equation. Determine weather the equation is ODE or PDE. Determine weather the equation is linear or nonlinear.

	Equation	Type		Linearity		Order
1	A simple model for the shape of a tsunami, or tidal wave, is given by $$\frac{dW}{dx}=W\sqrt{4-2W}$$	ODE	PDE	linear	nonlinear	
2	The model of the free pendulum given by $$\frac{d^2\theta}{dt^2}+2\lambda\frac{d\theta}{dt}+\omega^2\sin\theta=0$$ Where λ,ω are constants.	ODE	PDE	linear	nonlinear	
3	The steady state heat conduction equation $$-\frac{\partial}{\partial x}(k_1\frac{\partial u}{\partial x})-\frac{\partial}{\partial y}(k_2\frac{\partial u}{\partial y})=-\frac{\partial\rho}{\partial x}$$ Where k_1 and k_2 are constants.	ODE	PDE	linear	nonlinear	
4	The equation of a vibrating membrane $$\frac{\partial^2 u}{\partial t^2}-a^2(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2})=0$$ Where a is a constant.	ODE	PDE	linear	nonlinear	
5	The current $i(t)$ in an LRC series circuit satisfies $$L\frac{d^2i}{dt^2}+R\frac{di}{dt}+\frac{1}{C}i=E'(t)$$ Where L,R,C are constants.	ODE	PDE	linear	nonlinear	

(Q12) The following IVP

$$X' = \begin{bmatrix} \dfrac{1}{\sqrt{10-t}} & 5 \\ t^2 & \sin t \end{bmatrix} X + \begin{bmatrix} \dfrac{t}{\sin t} \\ \ln(t+50) \end{bmatrix}$$

$$X(0.1) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

has a unique solution on the interval

(a) $(0,10)$

(b) $(0,+\infty)$

(c) $(0,4)$

(d) $(-10,10)$

(e) $(0,3)$

(Q13) The general solution for the DE

$$y''-y=0$$

(a) $y=c_1e^{2x}+c_2e^{-2x}$

(b) $y=c_1\sin x+c_2\cos x$

(c) $y=c_1\sinh x+c_2\cosh x$

(Q14) Given that $y_1 = x^{-1}$, $y_2 = x^4$ are solutions for the DE

$$x^2 y'' - 2xy' - 4y = 0 \qquad (*)$$

and $y_p = -1$ is a particular solution for

$$x^2 y'' - 2xy' - 4y = 4 \qquad (**)$$

Then the solution for the IVP

$$x^2 y'' - 2xy' - 4y = 4$$
$$y(1) = 2, y'(1) = 2$$

(circle the correct answer)

(a) $y = 2x^{-1} + x^4$

(b) $y = 2x^{-1} + x^4 - 1$

(c) $y = x^{-1} + 2x^4 - 1$

(Q15) True or False

(a)	The point $x=2$ is a regular singular point of the DE $(x^2-4)y''+3(x-2)y'+5y=0$	**T**	**F**
(b)	The differential operator $[D^2-2D+2]^2$ annihilates the function $xe^x(\sin x+\cos x)$	**T**	**F**
(c)	$x=-2$ is an ordinary point of the DE $3x(x+2)y''+(x-2)y'-y=0$	**T**	**F**
(d)	$(\frac{1}{e^{i\beta}})^2=\cos(2\beta)-i\sin(2\beta)$	**T**	**F**
(e)	The DE: $x^2y''-xy'+2y=0$ is a Cauchy-Euler equation.	**T**	**F**
(f)	If $\Phi(t)$ is a fundamental matrix of the system $X'=AX$,then $\Phi'(t)=A\Phi(t)$.	**T**	**F**
(g)	We can not find two linearly independent solutions of the DE $xy''+y=0$ in the form of a power series centered at $x=2$.	**T**	**F**
(h)	$\left.\begin{array}{l} x^2y''-xy'+2y=0 \\ y(0)=1, y(1)=0 \end{array}\right\}$ is a Boundary Value Problem (BVP)	**T**	**F**
(i)	A constant multiple of a solution of a homogeneous linear differential equation is also a solution.	**T**	**F**
(j)	A set of functions is linearly dependent if at least one function can be expressed as a linear combination of the remaining functions.	**T**	**F**
(k)	If $y_1=1, y_2=x^2$ then $W(y_1,y_2)=-2x$	**T**	**F**

(Bonus 1) Consider the DE:

$$(x+2)y''+\frac{1}{x-5}y'+\frac{x-2}{(x^2+4)(x^2-6x+14)}y=0$$

Without solving the DE, find a lower bound for the radius of convergence of power series solutions about the point $x = 2$. The pwer series solutions centered at $x = 2$ will converge at least for $|x-2| < R$. Then $R =$

Show all your work then Circle your answer

(a) 3

(b) 2

(c) $2\sqrt{2}$

(d) $2\sqrt{3}$

(e) 4

(f) $2\sqrt{1.5}$