King Fahd University of Petroleum and Minerals
Department of Mathematical Sciences
Math 202 Exam I
Semester II, 2008- (072)
Dr. Faisal Fairag

Serial NO:		
ID:		
Name:	KEY	FORM **A**

Q		Points
1		30
2		30
3		20
4		15
5		15
6		20 (5 each)
Bonus		20
Total		130

(You are an **A+** student when you believe you are an **A+** student)

☺ Say a prayer & Good luck ☺

1) Solve the given initial value problem (Note: it is **separable** DE) (#7/p54)

$$\frac{dy}{dx} = e^{4x-2y}$$
$$y(0) = 0$$

$$\Rightarrow \quad dy = e^{4x-2y}\,dx$$

$$dy = e^{4x} \cdot e^{-2y}\,dx$$

$$e^{2y}\,dy = e^{4x}\,dx \quad 6$$

integrate both sides

$$\int e^{2y}\,dy = \int e^{4x}\,dx \quad 6$$

$$\frac{1}{2}e^{2y} = \frac{1}{4}e^{4x} + c_1 \text{ ——— (1)} \quad 6$$

$$y(0)=0 \Rightarrow \quad \frac{1}{2}e^{2\cdot 0} = \frac{1}{4}e^{4\cdot 0} + c_1$$

$$\Rightarrow \frac{1}{2} = \frac{1}{4} + c_1 \Rightarrow c_1 = \frac{1}{2} - \frac{1}{4} = \frac{1}{4} \text{ ——— (2)} \quad 6$$

$$(1), (2) \Rightarrow \quad \frac{1}{2}e^{2y} = \frac{1}{4}e^{4x} + \frac{1}{4}$$

multiply by 4

$$\boxed{2e^{2y} = e^{4x} + 1} \quad 6$$

2) Solve the given initial value problem (Note: it is **linear**) (#11/p65)

$$x\frac{dy}{dx}+4y=x^3-x$$
$$y(1)=1$$

$$xy'+4y=x^3-x$$

divid by x $\quad (x\neq 0 \Rightarrow (0,+\infty))$

$$y'+\frac{4}{x}y=x^2-1 \quad \text{——— (1)} \qquad 6$$

it is linear with $P(x)=\frac{4}{x}$, $f(x)=x^2-1$

$$\text{integrating factor} = K(x) = e^{\int \frac{4}{x}dx} = e^{4\int\frac{dx}{x}} = e^{4\ln x} = e^{\ln x^4} = x^4 \qquad 6$$

multiply (1) by x^4

$$x^4y'+4x^3y = x^6-x^4$$

$$\frac{d}{dx}\left[x^4y\right] = x^6-x^4 \qquad 6$$

$$\Rightarrow x^4y = \int (x^6-x^4)\,dx$$

$$x^4y = \frac{1}{7}x^7-\frac{1}{5}x^5+C$$

$$\Rightarrow y=\frac{1}{7}x^3-\frac{1}{5}x+Cx^{-4}$$

$$y(1)=1 \Rightarrow 1=\frac{1}{7}-\frac{1}{5}+C \Rightarrow C=\frac{37}{35} \qquad 6$$

$\boxed{y=\frac{1}{7}x^3-\frac{1}{5}x+\frac{37}{35}x^{-4}}$ is the sol for IVP.

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3) Find values of m so that the function $y = x^m$ is a solution of the given differential equation: Explain your reasoning.

$$x^2 y'' - 7xy' + 15y = 0$$

(#28/p11)

$y = x^m \Rightarrow y' = m x^{m-1}$

$\Rightarrow y'' = m(m-1) x^{m-2}$ 6

$x^2 y'' - 7 x y' + 15 y = 0$

$x^2 \cdot m(m-1) x^{m-2} - 7x \cdot m x^{m-1} + 15 x^m = 0$

$m(m-1) x^m - 7 m x^m + 15 x^m = 0$

$[m(m-1) - 7m + 15] x^m = 0$ 6

$[m^2 - m - 7m + 15] x^m = 0$

$[m^2 - 8m + 15] x^m = 0$

$\Rightarrow m^2 - 8m + 15 = 0$

$(m - 3)(m - 5) = 0$

$\Rightarrow \boxed{m = 3, 5}$ 8

4) Consider the following IVP

$$xy' = y$$
$$y(2) = 0$$

$\Longrightarrow y' = \frac{y}{x}$

$f(x,y) = \frac{y}{x}$

$\frac{\partial f}{\partial y} = \frac{1}{x}$

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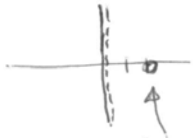
(a) The IVP has a unique solution.
(b) The IVP has no solution.
(c) The IVP has an infinite number of solutions.

f and $\frac{\partial f}{\partial y}$ con't xy-plane $x \neq 0$

$y(2)=0$



(2,0)

5) Consider the DE:

$$\frac{dy}{dx} = y^2 - 4$$

Which of the following statements is **FALSE**.

(a) $y = 2$ is a particular solution.
(b) $y = -2$ is a singular solution.
(c) $y = -2$ is a particular solution.
(d) $y = -3$ is a singular solution.

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6) TRUE or FALSE

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(a) The DE: $x^3 y''' + 2x^2 y'' - xy' + y = 12x^2$ is linear in y. (..T..)

(b) The order of the DE: $y' = \dfrac{1}{x - y''}$ is 1. (..F..)

(c) $y = 5\tan 5x$ is a solution for the DE $y' = 25 + y^2$. (..T..)

(d) $(y^2 + \sin y)xy' + y'\cos y = x$ is a linear DE in x. (..F..)

☺ Best Wishes ☺

Bonus) Give an example of a differential equation which satisfy the following conditions:

1) It is linear differential equation
2) It is separable
3) $y(x) = 3x$ is a solution.

Consider the following DE :

$$\boxed{y' = 3} \quad (*)$$

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or any other DE

$(*)$ is linear

$(*)$ is separable $\quad \frac{dy}{dx} = 3$

$dy = 3\,dx$

$(*)$ LHS $= y' = \frac{d}{dx}[3x] = 3 = $ RHS

~~RHS~~ $\Rightarrow y(x) = 3x$ is a solution.