Sec: 3 , 4 ID: **Quiz (5,6)** Name:

Consider the differential equation :

$$x^2y'' + xy' + (x^2 - \frac{1}{4})y = 0$$

This equation occur frequently in advanced studies in applied mathematics, physics, and engineering. It is called Bessele's equation

(1) Show that $x = 0$ is a singular point.

(2) Show that $x = 0$ is a regular singular point.

(3) Use the follwing formula $r(r-1) + p_0r + q_0 = 0$ to write the indicial equation.
The indicial equation is ....

(4) Find r_1 and r_2 (indicial roots) $r_1 =$ ..... $r_2 =$ ..... $r_1 - r_2 =$ ....

(5) Let $r = r_1$ and $y = \sum_{n=0}^{\infty} c_n x^{n+r}$ and write the recrrance relation.
the recrrance equation is ....

(6) Use the recrrance relation in part (5) to write c_{2n} and c_{2n+1}.

$$c_{2n} = \text{..............................} \quad n = 0, 1, 2, ...$$
$$c_{2n+1} = \text{..............................} \quad n = 0, 1, 2, ...$$

(7) Use c_{2n} and c_{2n+1} in part (6) to write the first solution $y_1(x)$.
$y_1(x) =$ ....

(8) Find the interval of convergence of $y_1(x)$
$y_1(x)$ valid in ....$< x <$.............

(9) Use the following formula $y_2 = y_1 \int \frac{K(x)}{y_1^2} dx$ to find the second linearly independent solution.
$y_2(x) =$ ....

(10) (Alternative method to find $y_2(x)$)
Since $r_1 - r_2 =$ positive integer, Assume $y_2 = cy_1(x)\ln x + \sum_{n=0}^{\infty} b_n x^{n+r_2}$, determine coefficients b_n and c to write the second solution $y_2(x)$.
$y_2(x) =$...

(11) Show that $y_2(x)$ obtained in part (9) is equivalent to $y_2(x)$ obtained in part (10)
(Hint: read the subsection titled Equivalent solution in page 302).

(12) Solve the differential equation subject to $y(\frac{\pi}{4}) = 0, y'(\frac{\pi}{4}) = \frac{4}{\pi}$.

(13) Let $\hat{y}$ be the solution of IVP obtained in part (12). Compute $\hat{y}(\frac{\pi}{6})$.

Show all your work in seperate pages - submit this page and all other pages Monday 3/5/99