King Fahd University of Petroleum and Minerals
Department of Mathematical Sciences

Math 202

Second Exam

Semester I, 1998–99 (981)

Name: ______________________________ ID #: ________________

Section #: ______________________________

Notes

- You must show all your work to justify your answer.
- Be as organized as possible.

Problem	
1	
2	
3	
4	
5	
6	
7	
Total:	

Problem # 1. (10 points)

Given that $y_1 = x^3 \ln x$ is a solution of the homogeneous differential equation

(1) $\quad x^2y'' - 5xy' + 9y = 0$

(a) Find a second solution y_2 of equation (1). (4 points)

(b) Solve the following differential equation subject to the initial conditions (4 points)

$$x^2y'' - 5xy' + 9y = x^{-4} \qquad y(1) = 0,\ y'(1) = 1.$$

(c) Find an interval around $x = 1$ for which the above initial value problem in part (b) has a unique solution. (2 points)

Problem # 2. (10 points)

Solve the given differential equation subject to the indicated initial conditions

$$y'' + y = 8\cos 2x - 4\sin x, \qquad y\left(\frac{\pi}{2}\right) = -1, \ \ y'\left(\frac{\pi}{2}\right) = 0.$$

[Use undetermined coefficients–Annihilator approach].

Problem # 3. (10 points)

Solve the given system subject to the initial conditions

$$\begin{aligned} 2x' + y' &= y + t \\ x' + y' &= t^2; \qquad x(0) = 1,\ y(0) = 1. \end{aligned}$$

Problem # 4. (10 points)

(a) Solve the following differential equation by variation of parameters. (8 points)

$$y''' + y' = \tan x$$

(b) State an interval on which the general solution of the above differential equation is defined. (2 points)

Problem # 5. (10 points)

(a) Solve the following differential equation subject to the initial conditions. (7 points)

$$x^3y''' + 3x^2y'' + 2xy' = 0 \quad y(1) = 5,\ y'(1) = 3,\ y''(1) = 1.$$

(b) Find the general solution of the following differential equation (3 points)

$$(D^4 + 2D^2 + 1)(D^2 - 2D + 1)y = 0.$$

Problem # 6. (10 points)

(a) Obtain the Wronskian of the functions (3 points)

$$y_1 = 1, \quad y_2 = x, \quad y_3 = \frac{x^4}{12} - \frac{x^2}{2}.$$

(b) Are the functions $y_1(x), y_2(x), y_3(x)$ linearly dependent or linearly independent? (use the result from part (a)). (2 points)

(c) Given that $y_{p_1} = e^x + e^{-x}$ and $y_{p_2} = e^x - e^{-x}$ are particular solutions of $xy'' + 2y' - xy = 2e^x - 2e^{-x}$ and $xy'' + 2y' - xy = 2e^x + 2e^{-x}$, respectively, find particular solutions of $xy'' + 2y' - xy = 3e^x + 5e^{-x}$. (5 points)

Problem # 7. (10 points)

Find the general solution of the following differential equation

$$2y'' - xy''' + 12x^3 = 0.$$