Math 202

(Selected H W Problems) (4.7)

10) $4x^2y'' + 4xy' - y = 0$

$y = x^m$ will lead to auxiliary eqn

$4m(m-1) + 4m - 1 = 0$

or $4m^2 - 4m + 4m - 1 = 0$

$\Rightarrow 4m^2 - 1 = 0 \Rightarrow m = \pm \frac{1}{2}$

$y = c_1 x^{-1/2} + c_2 x^{1/2}$.

16) $x^3y''' + xy' - y = 0$

$y = x^m$ will lead to auxiliary eqn

$m(m-1)(m-2) + m - 1 = 0 \Rightarrow$

$m^3 - 3m^2 + 2m + m - 1 = 0$

or $m^3 - 3m^2 + 3m - 1 = 0$

$m = 1$ is a root. Use long division by $m-1$ to obtain

$(m-1)(m^2 - 2m + 1) = 0$

$\Rightarrow (m-1)(m-1)(m-1) = 0$

$m = 1, 1, 1$ (repeated)

$y = c_1 x + c_2 x \ln x + c_3 x (\ln x)^2$.

19) $xy'' - 4y' = x^4 \Rightarrow x^2y'' - 4xy' = x^5$

Step 1 Homogeneous D.E

$x^2y'' - 4xy' = 0$

$y = x^m$ leads to $m(m-1) - 4m = 0$

or $m^2 - 5m = 0 \Rightarrow m(m-5) = 0$

$m = 0, 5$

$y_1 = 1, \; y_2 = x^5$

$y_c = c_1 + c_2 x^5$.

Step 2 To apply variation of parameter method, we must first put equation in form

$y'' + P(x)y' + Q(x)y = f(x)$

Thus we re-write equation as

$y'' - \frac{4}{x}y' = x^3$ (Divide by coeff of y'')

Now we find $\boxed{f(x) = x^3}$

$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} 1 & x^5 \\ 0 & 5x^4 \end{vmatrix} = 5x^4$

$W_1 = \begin{vmatrix} 0 & x^5 \\ x^3 & 5x^4 \end{vmatrix} = -x^8$

$W_2 = \begin{vmatrix} 1 & 0 \\ 0 & x^3 \end{vmatrix} = x^3$

$u_1' = \frac{W_1}{W} = \frac{-x^8}{5x^4} = -\frac{1}{5}x^4$

$u_1 = -\frac{1}{5(5)}x^5 = -\frac{1}{25}x^5$ ...

$u_2' = \frac{W_2}{W} = \frac{x^3}{5x^4} = \frac{1}{5x}$

$u_2 = \frac{1}{5}\ln x$ ...

Thus $y_p = u_1y_1 + u_2y_2$

$= -\frac{1}{25}x^5(1) + \frac{1}{5}x^5 \ln x$.

(already in y_c)

$y = y_c + y_p = c_1 + c_2x^5 + \frac{1}{5}x^5 \ln x$

39) $(x+2)^2y'' + (x+2)y' + y = 0$

We can put $x + 2 = t$, $\frac{dt}{dx} = 1$

$\frac{dy}{dx} = \frac{dy}{dt}\frac{dt}{dx} = \frac{dy}{dt}$

$\frac{d^2y}{dx^2} = \frac{d^2y}{dt^2}$

So D.E $\Rightarrow$

$t^2\frac{d^2y}{dt^2} + t\frac{dy}{dt} + y = 0$

which can now be solved.