King Fahd University of Petroleum and Minerals
Department of Mathematical Sciences
Math 101 Final Exam Semester I, 2003--2004(031) Dr. Faisal Fairag

Multiple Choice Answers (Fill the Correct Circle)

	a	b	c	d	e
4	○	○	○	○	●
5	○	○	○	●	○
6	○	●	○	○	○
7	○	○	○	●	○
8	○	○	○	○	●
9	○	○	○	○	●
10	○	●			
11	○	○	○	○	●
12	○	○	○	●	○
13	●	○	○	○	

Multiple Choice Answers (Fill the Correct Circle)

	a	b	c	d	e
14	●	○	○	○	
15	●	○	○	○	
16	●	○	○	○	
17		○	●	○	
18	○	●	○	○	
19	●	○	○	○	
20	○	○	●	○	
21	○	○	●	○	
22	○	●	○	○	
23	○	○	●	○	
24	○	●	○	○	○
25	○	○	●	○	○
26	○	○	○	○	●
27	●	○	○	○	○
28	○	●	○	○	○
29	●	○			
30	●	○			
31	○	●	○	○	○
32		●	○	○	○
33	○	○	●	○	○

Name: KEY

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Section: 11 28

Question #		Points
1		35
2		30
3		35
4-13		5 each
14-33		10 each
TOTAL		

1. Use Newton's Method to approximate a solution of the equation $x^4 + x - 3 = 0$ (Use $x_1 = -1$ to find x_2, x_3, x_4, x_5, x_6).

$$f(x) = x^4 + x - 3 \quad , \quad f'(x) = 4x^3 + 1$$ 2

Newton's method

$$x_{n+1} = x_n - \frac{x_n^4 + x_n - 3}{4x_n^3 + 1}$$ 3

$$x_1 = -1.000$$

6 $$x_2 = -1 - \frac{1-1-3}{-4+1} = -2.000$$

6 $$x_3 = -2 - \frac{16-2-3}{-32+1} = -2 - \frac{11}{-31} = -1.6451613$$

6 $$x_4 = -1.6451613 - \frac{2.6802825}{-16.81088} = -1.4857239$$

6 $$x_5 = -1.4857239 - \frac{0.38678339}{-12.11820} = \underline{-1.45380640}$$

6 $$x_6 = -1.45380640 - \frac{0.01330022}{-11.290787} = \underline{-1.45262844}$$

$$x_7 = -1.45262844 - \frac{0.000017587}{-11.2609358} = \underline{-1.452628}$$

2. Find $\lim_{x\to+\infty}(\sqrt{x^2+x}-x)$.

$$\lim_{x\to+\infty}\left(\sqrt{x^2+x}-x\right) =$$

6

$$= \lim_{x\to+\infty} x\left(\frac{\sqrt{x^2+x}}{x}-1\right) = \lim_{x\to+\infty} x\left(\sqrt{1+\tfrac{1}{x}}-1\right)$$

of type $0\cdot\infty$

6 6

$$= \lim_{x\to+\infty} \frac{\sqrt{1+\frac{1}{x}}-1}{1/x}$$ of type $\frac{0}{0}$ (apply L'H)

6

$$= \lim_{x\to+\infty} \frac{\frac{-1/x^2}{2\sqrt{1+\frac{1}{x}}}}{-1/x^2} = \lim_{x\to+\infty} \frac{1}{2\sqrt{1+\frac{1}{x}}}$$

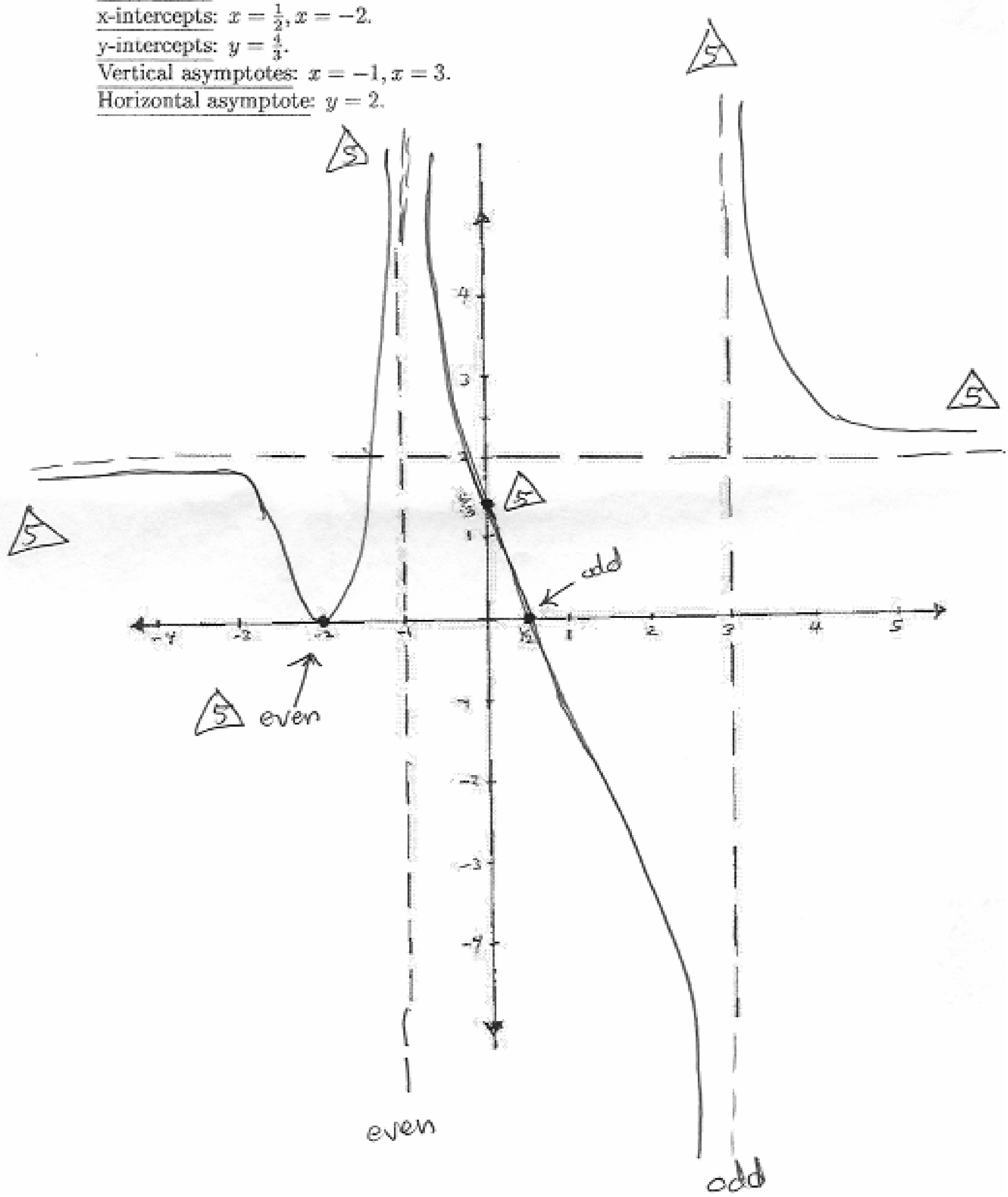
$$= \frac{1}{2\sqrt{1+0}} = \frac{1}{2}$$

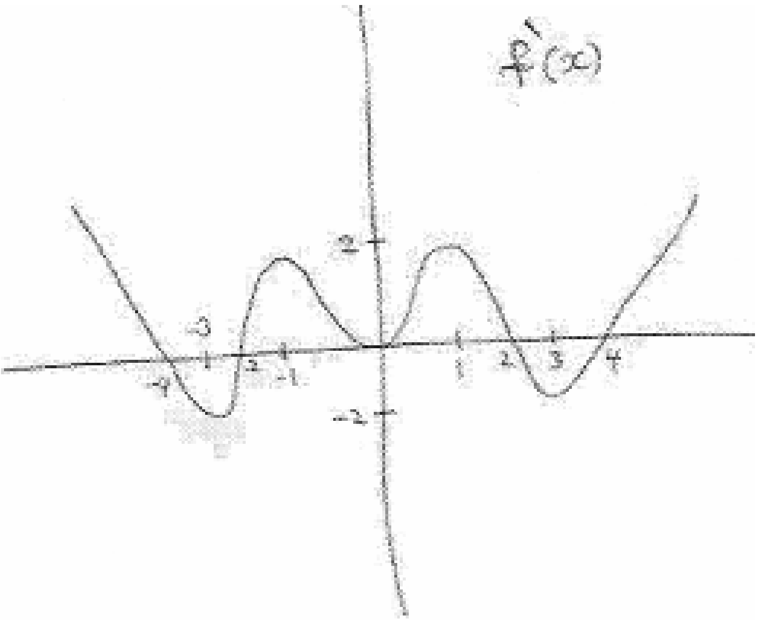
6

Now $\lim_{x\to+\infty}\left(\sqrt{x^2+x}-x\right) = \frac{1}{2}$

3. Sketch a graph of $f(x) = \frac{(2x-1)(x+2)^2}{(x+1)^2(x-3)}$.

<u>Symmetries</u>: There are no symmetries.
<u>x-intercepts</u>: $x = \frac{1}{2}, x = -2$.
<u>y-intercepts</u>: $y = \frac{4}{3}$.
<u>Vertical asymptotes</u>: $x = -1, x = 3$.
<u>Horizontal asymptote</u>: $y = 2$.





4. $f(x)$ has __________________ critical numbers

(a) 1

(b) 2

(c) 3

(d) 4

(e) 5

e

5. $f(x)$ has __________________ relative extrema

(a) 1

(b) 2

(c) 3

(d) 4

(e) 5

d

6. $f(x)$ is increasing in the interval

(a) $(-\infty, 0)$

(b) $(0, 2)$

(c) $(2, +\infty)$

(d) $(1, 3)$

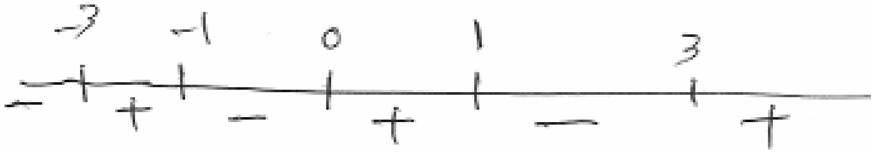
(e) $(-\infty, +\infty)$

b

7. $f(x)$ is concave down in the interval

d

(a) $(-\infty, -2)$
(b) $(0, 2)$
(c) $(2, 4)$
(d) $(-1, 0)$
(e) $(-\infty, +\infty)$



8. $f(x)$ has ________________ inflection points

(a) 1
(b) 2
(c) 3
(d) 4
(e) 5

e

9. $f(x)$ has ________________ stationary points

(a) 1
(b) 2
(c) 3
(d) 4
(e) 5

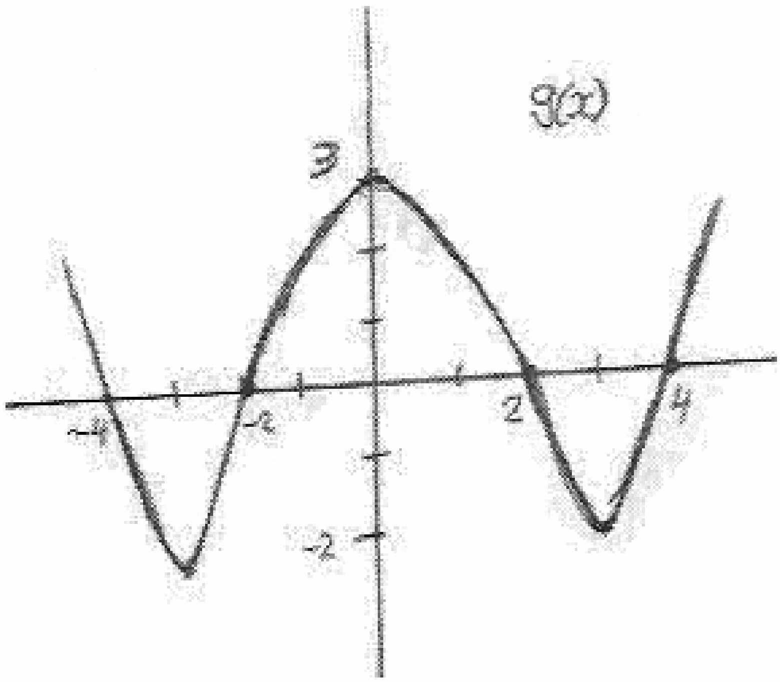
e

10. $f(x)$ has absolute minimum

(a) True
(b) False

b

In questions 11-13, consider the adjacent graph of $g(x)$.



11. $\lim_{x \to 0^+} g(x) =$

    (a) $+\infty$

    (b) 0

    (c) 1

    (d) 2

    (e) 3

e

12. $\lim_{x \to +\infty} g(x) =$

    (a) $-\infty$

    (b) 0

    (c) 4

    (d) $+\infty$

    (e) 1

d

13. $\lim_{h \to 0} \dfrac{g(3+h) - g(3)}{h}$

    (a) 0

    (b) 1

    (c) $\dfrac{3+h}{h}$

    (d) $\dfrac{h-3}{h}$

a

In questions 14-17, consider the function $f(x) = x^3 - 3x^2$.

$f'(x) = 3x^2 - 6x$

$f''(x) = 6x - 6$

14. $f(x)$ is concave up on

   (a) $(1, +\infty)$
   (b) $(-\infty, 1)$
   (c) $(-3, 3)$
   (d) $(0, +\infty)$

a

15. $f(x)$ has inflection point at $x =$

   (a) 1
   (b) 2
   (c) 3
   (d) 4

a

$f'(x) = 3x(x-2)$

16. $f(x)$ has relative maximum at $x =$

   (a) 0
   (b) 1
   (c) 2
   (d) 3

a

$f''(x) = 6(x-1)$

17. $f(x)$ has relative minimum at $x =$

   (a) 0
   (b) 1
   (c) 2
   (d) 3

c

$+ \quad - \quad +$

$0 \qquad 2$

18. $f(x) = x^2 - 4x + 3$ is increasing on

$f'(x) = 2x - 4 = 2(x-2)$

(a) $(-\infty, 2)$
(b) $(2, +\infty)$
(c) $(0, 10)$
(d) $(0, +\infty)$

b

19. $f(x) = x^2 - 4x + 5$ has absolute maximum on the on the interval $[0, 3]$ at $x =$

(a) 0
(b) 1
(c) 2
(d) 3

$f'(x) = 2x - 4 = 2(x-2)$

0 , 2 , 3

$f(0) = 5$, $f(2) = 1$, $f(3) = 2$

a

20. The function $f(x) = x^{1/3} + (x-3)^{-1}$ satisfies the hypotheses of the mean value theorem on the interval

$f'(x) = \frac{1}{3} x^{-2/3} - (x-3)^{-2} = \frac{1}{3} \frac{1}{x^{2/3}} - \frac{1}{(x-3)^2}$

(a) $[2, 4]$ ×
(b) $[-1, 1]$ ×
(c) $[1, 2]$
(d) $[-1, 4]$ ×

c

21. The value of c in the interval $[1, 2]$ that satisfies the condition of the Mean Value Theorem is (where $f(x) = x^2 - 1$)

$f'(x) = 2x$

(a) $5/3$
(b) 3
(c) $3/2$
(d) $5/4$

$$\frac{f(b) - f(a)}{b - a} = \frac{f(2) - f(1)}{2 - 1} = \frac{3 - 0}{2 - 1} = \frac{3}{1} = 3$$

$f'(c) = 2c = 3 \Rightarrow c = 3/2$ c

22. $\lim_{x \to 0^+} \left(\ln x - \frac{1}{x} \right)$

(a) $+\infty$
(b) $-\infty$
(c) e^{-2}
(d) $-1/2$

b

$\lim_{x \to 0^+} (\ln x) = -\infty$ and $\lim_{x \to 0^+} \frac{1}{x} = +\infty$

Hence $\lim_{x \to 0^+} (\ln x - \frac{1}{x}) = (-\infty) - (+\infty) = -\infty$

23. $\lim_{x \to +\infty} (2xe^{-x})$ of type $0 \cdot \infty$

(a) 2
(b) $+\infty$
(c) 0
(d) e^2

$\lim_{x \to +\infty} (2xe^{-x}) = \lim_{x \to +\infty} \frac{2x}{e^x}$ of type ∞/∞

apply L'H $= \lim_{x \to +\infty} \frac{2}{e^x} = 0$

c

24. If $f(x) = tan^{-1}(x^4)$ then $f''(1) =$

(a) $-\frac{1}{4}$

(b) $-\frac{1}{2}$

(c) 0

(d) $\frac{1}{2}$

(e) $\frac{1}{4}$

b

$f'(x) = \frac{4x^3}{(x^4)^2+1} = \frac{4x^3}{x^8+1}$

$f''(x) = \frac{12x^2(x^8+1) - 8x^7(4x^3)}{(x^8+1)^2}$

$f''(1) = \frac{12(2) - 8(4)}{4} = \frac{24-32}{4} = -2$

25. Use local linear approximation to approximate $(1.001)^{37} \approx$

(a) 1.0376

(b) 1.03767

(c) 1.037

(d) 1.038

(e) 1.036

c

$f(x) = x^{37} \Rightarrow f'(x) = 37x^{36}$

$x_0 = 1 \Rightarrow f(1) = 1 \Rightarrow$ point $(1,1)$

$f'(1) = 37 \Rightarrow$ slope $= 37$

$l(x) - 1 = 37(x-1) \Rightarrow l(x) = 1 + 37(x-1)$

$l(1.001) = 1 + 37(.001)$

26. Use local linear approximation to approximate $\frac{1}{\sqrt{3.9}} \approx$

(a) 0.506371

(b) 0.5125

(c) 0.51256

(d) 0.4875

(e) 0.50625

e

$f(x) = \frac{1}{\sqrt{x}}$, $f'(x) = -\frac{1}{2}x^{-3/2} = -\frac{1}{2}\frac{1}{(\sqrt{x})^3}$

$x_0 = 4 \Rightarrow f(4) = \frac{1}{2}$, slope $= f'(4) = -\frac{1}{2}\frac{1}{8} = -\frac{1}{16}$

$l(x) = \frac{1}{2} + (-\frac{1}{16})(x-4) \Rightarrow l(x) = \frac{1}{2} - \frac{1}{16}(x-4)$

$l(3.9) = \frac{1}{2} - \frac{1}{16}(-0.1) = 0.50625$

27. A spherical balloon is inflated so that its radius is increasing at the rate of 0.5 m/min. How fast is the volume of the balloon increasing when the radius is 2 m?
(Hint: volume of sphere $= \frac{4}{3}\pi r^3$).

(a) $8\pi \quad m^3/min$

(b) $8 \quad m^3/min$

(c) $16\pi \quad m^3/min$

(d) $16 \quad m^3/min$

(e) $\frac{32}{3}\pi \quad m^3/min$

a

$V = \frac{4}{3}\pi r^3$

$\frac{dv}{dt} = 4\pi r^2 \cdot \frac{dr}{dt}$

$= 4\pi(2)^2(0.5) = 8\pi$

28. The slope of the curve $y = sec^2 x$ at $x = 0$ equals

(a) -1

(b) 0

(c) 1

(d) $+\infty$

(e) $-\infty$

b

$y' = 2\sec x \cdot \sec x \cdot \tan x = \frac{2\sin x}{\cos^3 x}$

$y'(0) = \frac{2 \cdot \sin(0)}{\cos^3(0)} = \frac{0}{1} = 0$

29. If $\lim_{x \to 3} f(x)$ does not exist, then f is discontinuous at $x = 3$

(a) True

(b) False

a

30. A rational function is continuous at every number where the dominator is nonzero

(a) True

(b) False

a

31. Find the x-coordinate of the point at the graph of $y = x^4 + 2x^2$ where the tangent line is horizontal.

(a) -1

(b) 0

(c) 1

(d) 2

(e) 3

b

$$y' = 4x^3 + 4x$$
$$= 4x(x^2+1)$$
$$y' = 0 \Rightarrow x = 0$$

32. If $f(x) = \frac{lnx}{1+lnx}$, then $f'(1) =$

(a) 0

(b) 1

(c) e

(d) $-e$

(e) -1

b

$$f'(x) = \frac{\frac{1}{x}(1+\ln x) - \frac{1}{x}(\ln x)}{(1+\ln x)^2}$$
$$= \frac{\frac{1}{x}}{(1+\ln x)^2}$$
$$f'(1) = \frac{1}{(1+0)^2} = 1$$

33. If $f(x) = \frac{e^{3x}}{1+e^{-2x}}$, then $f'(0) =$

(a) $\frac{1}{2}$

(b) $\frac{1}{1+e}$

(c) 2

(d) $\frac{3e}{1+e}$

c

$$f'(x) = \frac{3e^{3x}(1+e^{-2x}) - (-2e^{-2x})e^{3x}}{(1+e^{-2x})^2}$$

$$f'(0) = \frac{3(1+1) - (-2)}{2^2} = \frac{6+2}{4} = \frac{8}{4} = 2$$