KEY SOLUTION (MCQ)

	FORM A	FORM B	FORM C	FORM D
1	A	A	E	E
2	E	E	D	C
3	E	E	D	C
4	A	A	E	E
5	C	D	C	D
6	D	C	C	D
7	D	D	A	A
8	C	D	A	A
9	D	C	D	D
10	E	C	C	E
11	C	E	E	C
12	D	D	D	D
13	A	A	A	A
14	A	A	B	B
15	B	B	D	E
16	B	B	B	E
17	B	B	A	A
18	A	A	B	B
19	E	D	E	D
20	D	E	B	B
21	D	D	A	B
22	E	B	D	A
23	b	E	E	D

EXAM (1) FORM A

Name: ID # Sec # (11) (28)

24) $f(x) = \begin{cases} k^2x+1 & x>1 \\ k+1 & x=1 \\ 3kx-1 & x<1 \end{cases}$

(a) Find the values of k so that the function f has a removable discontinuity at x=1 $\boxed{k=2}$
(SHOW ALL YOUR WORK)

2 $\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} [k^2x+1] = k^2+1$

2 $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} [3kx-1] = 3k-1$

$\Rightarrow k^2+1 = 3k-1$
$k^2-3k+2=0$
$(k-2)(k-1)=0$
$\Rightarrow k=1, 2$ 2

Case1: $k=1 \Rightarrow f(x) = \begin{cases} x+1 & x>1 \\ 2 & x=1 \\ 3x-1 & x<1 \end{cases} \Rightarrow \lim_{x\to 1} f(x) = 2$; $f(1)=2$

Now, f(x) is cont at $x=1$ 6

Case2: $k=2 \Rightarrow f(x) = \begin{cases} 4x+1 & x>1 \\ 3 & x=1 \\ 6x-1 & x<1 \end{cases} \Rightarrow \lim_{x\to 1} f(x) = 5$, $f(1)=3$

Now, f(x) has a removable discont. at $x=1$ 6

$\boxed{\text{Answer: } k=2}$ 2

(b) Find the values of k so that the function f is continuous from the left at x=1
(SHOW ALL YOUR WORK)

2 $\lim_{x\to 1^+} f(x) = \lim_{x\to 1^-} (3kx-1) = 3k-1$

2 $f(1) = k+1$

f cont from the left $\Rightarrow f(1) = \lim_{x\to 1^-} f(x)$

2 $k+1 = 3k-1 \Rightarrow 2 = 2k$

$\Rightarrow k=1$

$\boxed{\text{Answer: } k=1}$ 2

25) State the intermediate value theorem.

If f is cont. on $[a,b]$ and k is any number between $f(a), f(b)$ then there is at least one number x in $[a,b]$ such that $f(x) = k$

26) Find $\lim_{x \to -\infty} \frac{x^4 + x^2 + 3x}{x+1}$ **(SHOW ALL YOUR WORK)**

$$\lim_{x \to -\infty} \frac{\frac{x^4}{x} + \frac{x^2}{x} + \frac{3x}{x}}{\frac{x}{x} + \frac{1}{x}} = \lim_{x \to -\infty} \frac{x^3 + x + 3}{1 + \frac{1}{x}}$$

7

$$= \frac{\lim_{x \to -\infty} (x^3 + x + 3)}{\lim_{x \to -\infty} 1 + \lim_{x \to -\infty} \frac{1}{x}} = \frac{-\infty}{1 + 0}$$

3

$$= \frac{-\infty}{1} = -\infty$$

4

27) Find $\lim_{h \to 0} \frac{|2+h| - 2}{h}$ **(SHOW ALL YOUR WORK)**

$$\lim_{h \to 0^+} \frac{|2+h| - 2}{h} = \lim_{h \to 0^+} \frac{(2+h) - 2}{h} \quad (h > 0)$$

6

$$= \lim_{h \to 0^+} \frac{h}{h} = 1$$

$$\lim_{h \to 0^-} \frac{|2+h| - 2}{h} = \lim_{h \to 0^-} \frac{(2+h) - 2}{h} \quad (h < 0)$$

6

$$= \lim_{h \to 0^-} \frac{h}{h} = 1$$

$$\Longrightarrow \lim_{h \to 0} \frac{|2+h| - 2}{h} = 1$$

2