

College of Sciences	King Fahd University of Petroleum & Minerals	Department of Mathematical Sciences
Name: KEY	Quiz-5 MATH 101	ID: KEY SEC: 11 28

(Success is a Journey). Say a prayer and START

1) $\triangle 15$

Find $\lim_{x \rightarrow +\infty} \frac{\ln(x)}{x}$

of type ∞/∞ $\triangle 3$

$\triangle 4$ Apply L'H

$$\lim_{x \rightarrow +\infty} \frac{\ln(x)}{x} = \lim_{x \rightarrow +\infty} \frac{1/x}{1} = \lim_{x \rightarrow +\infty} \frac{1}{x} = 0 \quad \triangle 8$$

2) $\triangle 15$

Find $\lim_{x \rightarrow 0} (e^x + x)^{\frac{1}{x}}$

of type 1^∞

Let $y = \lim_{x \rightarrow 0} (e^x + x)^{\frac{1}{x}}$ $\triangle 2$

take \ln $\ln y = \lim_{x \rightarrow 0} \left[\frac{1}{x} \ln(e^x + x) \right]$

$$= \lim_{x \rightarrow 0} \left[\frac{\ln(e^x + x)}{x} \right] \text{ of type } \frac{0}{0} \quad \triangle 2$$

$$= \lim_{x \rightarrow 0} \left[\frac{e^x + 1}{e^x + x} \right] = \lim_{x \rightarrow 0} \frac{e^x + 1}{e^x + x} \quad \triangle 2$$

$$= \frac{2}{1} = 2 \Rightarrow \boxed{y = e^2}$$

Hence, $\lim_{x \rightarrow 0} (e^x + x)^{\frac{1}{x}} = e^2$ $\triangle 9$

3) Find all relative extrema and all inflection points.

$$f(x) = \frac{x^2}{x^2+1}, \quad f'(x) = \frac{2x(x^2+1) - 2x(x^2)}{(x^2+1)^2} = \frac{2x}{(x^2+1)^2}$$

$$f''(x) = \frac{2(x^2+1)^2 - 2(x^2+1)(2x)(2x)}{(x^2+1)^4}$$

$$= \frac{2(x^2+1)[x^2+1 - 4x^2]}{(x^2+1)^4} = \frac{2(x^2+1)(1-3x^2)}{(x^2+1)^4}$$

$$\boxed{f''(x) = \frac{2(1-\sqrt{3}x)(1+\sqrt{3}x)}{(x^2+1)^3}}$$

$$\boxed{f'(x) = \frac{2x}{(x^2+1)^2}}$$

$f'(x) = 0 \Rightarrow x = 0$ is critical number.

$$f''(0) = \frac{2(1)(1)}{1} = 2$$

\Rightarrow f has relative min at $x=0$

$$f''(x) = 0 \Rightarrow x = \frac{1}{\sqrt{3}}, \quad x = -\frac{1}{\sqrt{3}}$$

	$-\frac{1}{\sqrt{3}}$	0	$\frac{1}{\sqrt{3}}$
$2(1-\sqrt{3}x)$	+	+	-
$(1+\sqrt{3}x)$	-	+	+
$(x^2+1)^3$	+	+	+
$f''(x)$	-	+	-

Hence $x = \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}$ are inflection points

