College of Sciences	King Fahd University of Petroleum & Minerals	Department of Mathematical Sciences	
Name: KEY	**Bonus Quiz Form A** **MATH 101**	ID: KEY	SEC: **11 28**

(Success is a Journey). Say a Bismillah and START

10 each

1) The oblique asymptote of $f(x) = \frac{2x^3-3x+4}{x^2}$ is

A a) y=2x b) y=3x+4 c) y=2x-3 d) y=2x+3

2) Let $f(x)=4x^{1/3}-x^{4/3}$ There is a vertical tangent line and cusp at x=

a) 0 b) 1 c) 4/3 d) 4 A e) no cusp

3) Let $f(x)=4x^3-3x^4$ on $(-\infty,+\infty)$ Then f has absolute max at x =

A a) 1 b) 0 c) 5 d) 12 e) no absolute max

4) If we use Newton method to approximate the real solution of $x^3-x-1=0$ Then $x_4=$ (Hint: use $x_1=1$).

a) 1.3478 b) 1.3252 A c) 1.5 d) 1.345 e) 1.425

5) Number of critical numbers of the function $f(x)=\frac{x^2+1}{x}$ is

a) 0 b) 1 A c) 2 d) 3 e) 4

6) A cylindrical can, open at the top, is to hold 125π cm^3 of liquid. Then the radius that minimize the amount of material needed to manufacture the can is

A a) 5 cm b) 5π cm c) $5/\pi$ cm d) 25 cm

7) True or False :

5 each

1. The hypotheses of Rolle's Theorem are satisfied for $f(x)=x^2-6x+8$ on $[2,5]$ (T) (F)
2. The hypotheses of Rolle's Theorem are satisfied for $f(x)=\frac{x^2-1}{x+2}$ on $[-1,1]$ (T) (F)
3. The hypotheses of Mean Value Theorem are satisfied for $f(x)=\frac{x+1}{x-1}$ on $[-1,0]$ (T) (F)

8) Find all values of c in the interval [3,4] that satisfy the conclusion of the Mean Value Theorem.

$f(x) = x + \frac{1}{x}$, $f'(x) = 1 - \frac{1}{x^2}$ 4

$f(x)$ cont. on $[3,4]$ and diff on $(3,4)$

we need to find c such that

$$f'(c) = \frac{f(4)-f(3)}{4-3} = \frac{(4+\frac{1}{4})-(3+\frac{1}{3})}{1} = 1+\frac{1}{4}-\frac{1}{3}$$

4 $$= \frac{12+3-4}{12} = \frac{11}{12}$$

$$\Rightarrow 1 - \frac{1}{c^2} = \frac{11}{12} \Rightarrow \frac{1}{c^2} = 1 - \frac{11}{12} \Rightarrow \frac{1}{c^2} = \frac{1}{12}$$

$\Rightarrow c^2 = 12 \Rightarrow c = \pm\sqrt{12}$ but $-\sqrt{12} \notin [3,4]$

Hence, $\boxed{c = \sqrt{12}}$ 4

12

9) The equation $x^5 + x^4 - 5 = 0$ has one real solution. Approximate it by Newton's Method (use $x_1 = 1$).

$$f(x) = x^5 + x^4 - 5$$

$$f'(x) = 5x^4 + 4x^3$$

newton $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

6 $$x_{n+1} = x_n - \frac{x_n^5 + x_n^4 - 5}{5x_n^4 + 4x_n^3}$$ $n = 1, 2, 3, 4, \ldots$

$x_1 = 1.0000$

$x_2 = 1.3333$

$x_3 = 1.2394$

$x_4 = 1.2244$

13

① $\begin{array}{r} 2x \\ x^2 \overline{)\,2x^3 - 3x + 4} \\ \mp 2x^3 \\ \hline 0 - 3x + 4 \end{array} \Bigg\} \Rightarrow f(x) = 2x + \dfrac{-3x+4}{x^2}$

$\Rightarrow y = 2x$ is the oblique asympt.

---

② $f(x) = 4x^{1/3} - x^{4/3}$, $f'(x) = \frac{4}{3}x^{-2/3} - \frac{4}{3}x^{1/3}$

$= \frac{4}{3}\left(\dfrac{1-x}{x^{2/3}}\right)$

f is not diff at $x=0$

$$\left.\begin{array}{l} \lim\limits_{x\to 0^+} f'(x) = \lim\limits_{x\to 0^+} \frac{4}{3}\dfrac{(1-x)}{x^{2/3}} = +\infty \\ \lim\limits_{x\to 0^-} f'(x) = \lim\limits_{x\to 0^-} \frac{4}{3}\dfrac{(1-x)}{x^{2/3}} = +\infty \end{array}\right\} \Rightarrow \text{No cusp}$$

---

③ $$\left.\begin{array}{l} \lim\limits_{x\to +\infty} f(x) = \lim\limits_{x\to +\infty}(-3x^4) = -\infty \\ \lim\limits_{x\to -\infty} f(x) = \lim\limits_{x\to -\infty}(-3x^4) = -\infty \end{array}\right\} \Rightarrow \begin{array}{l}\text{No abs. min} \\ \text{there is abs. max}\end{array}$$

$f'(x) = 12x^2 - 12x^3 = 12x^2(1-x) \Rightarrow x=0,\ x=1$ critical

		0		1	
$12x^2$	+	0	+		+
$1-x$	+		+		—
$f'(x)$	+		+		—

Relative max

Hence, f has ~~Absoulte~~ max at $x=1$.

---

④ $f(x) = x^3 - x - 1$, $f'(x) = 3x^2 - 1 \Rightarrow x_{n+1} = x_n - \dfrac{x_n^3 - x_n - 1}{3x_n^2 - 1}$

$x_1 = 1.000$ $\qquad$ $x_3 = 1.3478$

$x_2 = 1.500$ $\qquad$ $x_4 = 1.3252$

(5) $f(x) = \dfrac{x^2+1}{x}$. domain of f is $\mathbb{R} - \{0\}$

$$f'(x) = \frac{2x(x) - (1)(x^2+1)}{x^2} = \frac{2x^2 - x^2 - 1}{x^2} = \frac{x^2-1}{x^2}$$

$$= \frac{(x-1)(x+1)}{x^2}$$

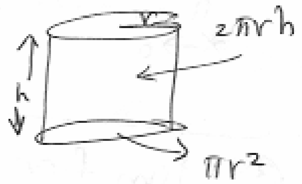
To find critical number we study $f'(x) = 0$ or $f'(x)$ not exist

$\Rightarrow$ $x = 1, -1$ and $x = 0$
But not in the domain

Hence we have only 2 critical numbers.

---

(6) r = radius of can, h = height of can, V = volume
S = matrial needed



$$\boxed{S = \pi r^2 + 2\pi r h}$$

$V = \pi r^2 h \Rightarrow 125\pi = \pi r^2 h$

$\Rightarrow h = \dfrac{125}{r^2}$

$$S(r) = \pi r^2 + 2\pi r\left(\frac{125}{r^2}\right) = \pi r^2 + \frac{2\pi(125)}{r}$$

$r \in (0, +\infty)$

Now, $\boxed{\text{Find Absoulte min for } S(r) = \pi r^2 + \frac{2\pi(125)}{r} \text{ on } (0, +\infty)}$

$\lim_{r \to 0^+} S(r) = +\infty$, $\lim_{r \to +\infty} S(r) = +\infty$

$S'(r) = 2\pi r - (2\pi(125))\dfrac{1}{r^2} = 0 \Rightarrow r^3 = 125 \Rightarrow r = \sqrt[3]{125}$

$r = 5$, $S''(r) = 2\pi + 4\pi(125)\dfrac{1}{r^3} \Rightarrow S''(5) = +$tive positive

S has Absoulte min at $r = 5$ cm.

---

(7) (1) False because $f(5) \neq 0$ (2) True, $f(-1) = 0$, $f(1) = 0$
f cont on $[-1,1]$, diff on $(-1,1)$

(3) f cont on $[-1,0]$, f diff on $(-1,0)$