CHAPTER 3
The Derivative

SECTION 3.1

3.1.1 Let $f(x) = \dfrac{1}{x^2}$;

(a) Find the average rate of change of y with respect to x over the interval $[2, 3]$.

(b) Find the instantaneous rate of change of y with respect to x at the point $x = 2$.

(c) Find the instantaneous rate of change of y with respect to x at a general point x_0.

(d) Sketch the graph of $y = f(x)$ together with the secant and tangent lines whose slopes are given by the results in parts (a) and (b).

3.1.2 Let $f(x) = x^2 + 1$.

(a) Find the average rate of change of y with respect to x over the interval $[-2, -1]$.

(b) Find the instantaneous rate of change of y with respect to x at the point $x = -2$.

(c) Find the instantaneous rate of change of y with respect to x at a general point x_0.

(d) Sketch the graph of $y = f(x)$ together with the secant and tangent lines whose slopes are given by the results in parts (a) and (b).

3.1.3 Let $f(x) = \dfrac{1}{x-2}$.

(a) Find the average rate of change of y with respect to x over the interval $[3, 5]$.

(b) Find the instantaneous rate of change of y with respect to x at the point $x = 3$.

(c) Find the instantaneous rate of change of y with respect to x at a general point x.

(d) Sketch the graph of $y = f(x)$ together with the secant and tangent lines whose slopes are given by the results in parts (a) and (b).

3.1.4 Let $f(x) = \dfrac{1}{x+1}$.

(a) Find the average rate of change of y with respect to x over the given interval $[1, 3]$.

(b) Find the instantaneous rate of change of y with respect to x at the point $x = 1$.

(c) Find the instantaneous rate of change of y with respect to x at the general point x_0.

(d) Sketch the graph of $y = f(x)$ together with the secant and tangent lines whose slopes are given by the results in parts (a) and (b).

3.1.5 Let $f(x) = \dfrac{2}{3-x}$.

(a) Find the slope of the tangent to the graph of f at a general point x_0 using the method of Section 3.1

(b) Use the result in part (a) to find the slope of the tangent at $x_0 = 1$.

3.1.6 Let $f(x) = \dfrac{3}{x-1}$.

(a) Find the slope of the tangent to the graph of f at a general point x_0 using the method of Section 3.1.

(b) Use the result in part (a) to find the slope of the tangent at $x_0 = 4$.

3.1.7 Let $f(x) = \dfrac{1}{x^2}$.

(a) Find the slope of the tangent to the graph of f at a general point x_0 using the method of section 3.1.

(b) Use the result in part (a) to find the slope of the tangent at $x_0 = -2$.

3.1.8 Let $f(x) = 3x^2$.

(a) Find the slope of the tangent to the graph of f at a general point x_0 using the method of section 3.1.

(b) Use the result in part (a) to find the slope of the tangent at $x = 3$.

3.1.9 A rock is dropped from a height of 144 feet and falls toward the earth in a straight line. In t seconds, the rock drops a distance of $s = 16t^2$ feet.

(a) What is the average velocity of the rock while it is falling?

(b) Use the method of 3.1 to find the instantaneous velocity of the rock when it hits the ground.

3.1.10 A rock is dropped from a height of 64 feet and falls toward the earth in a straight line. In t seconds, the rock drops a distance of $s = 16t^2$ feet.

(a) What is the average velocity of the rock while it is falling?

(b) Use the method of Section 3.1 to find the instantaneous velocity of the rock when it hits the ground.

3.1.11 A particle moves in a straight line from its initial position so that after t seconds, its distance is given by $s = t^2 + t$ feet from its initial position.

(a) Find the average velocity of the particle over the interval $[1, 3]$ seconds.

(b) Use the method of Section 3.1 to find the instantaneous velocity of the particle at $t = 1$ second.

3.1.12 A particle moves in a straight line from its initial position so that after t seconds, its distance is given by $s = \dfrac{t}{t+2}$ feet from its initial position.

(a) Find the average velocity of the particle over the interval $[2, 3]$ seconds.

(b) Use the method of Section 3.1 to find the instantaneous velocity of the particle at $t = 2$ seconds.

3.1.13 Let $f(x) = x^2$.

Use the method of Section 3.1 to show that the slope of the tangent to the graph of f at $x = x_0$ is $2x_0$.

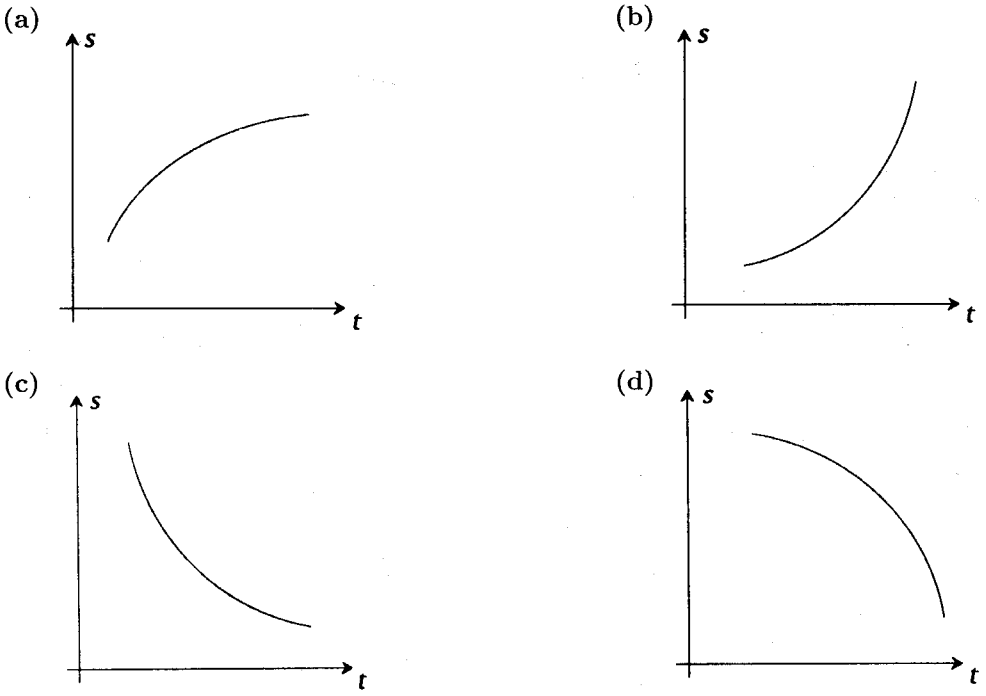
3.1.14 Let $f(x) = ax^2 + b$, where a and b are constants. Use the method of Section 3.1 to show that the slope of the tangent to the graph of f at $x = x_0$ is $2ax_0$.

3.1.15 Let $f(x) = ax^3 + b$, where a and b are constants. Use the method of Section 3.1 to show that the slope of the tangent to the graph of f at $x = x_0$ is $3ax_0^2$.

3.1.16 A particle moves in a straight line from its initial position so that after t seconds, its distance is given by $s = 16t^2$ feet. Use the method of Section 3.1 to show that the instantaneous velocity of the particle at $t = t_0$ seconds is $32t_0$.

3.1.17 A particle moves in a straight line from its initial position so that after t seconds, its distance is given by $s = 4 - 16t^2$ feet. Use the method of Section 3.1 to show that the instantaneous velocity of the particle at $t = t_0$ seconds is $v = -32t_0$.

3.1.18 The figure shows the position versus time curves of four different particles moving on a straight line. For each particle, determine if its instantaneous velocity is increasing or decreasing with time.



3.1.19 The figure shows the position versus time curve for a certain particle moving along a straight line. Estimate each of the following from the graph.

(a) The average velocity over the interval $0 \leq t \leq 4.6$

(b) The values of t at which the instantaneous velocity is zero

(c) The values of t at which the instantaneous velocity is maximum; minimum

(d) The instantaneous velocity when $t = 5$ seconds