Name: KEY Q2(A) Sec: 7, 11 ID: KEY

1) Let

$$f(x) = csc(\frac{1}{4}(2x^2+4)\pi)sec(\frac{1}{4}(2x^2+1)\pi)$$

then $f'(3) =$

$f'(x) = -\pi x \cot\left(\frac{\pi}{4}(4+2x^2)\right)\csc\left(\frac{\pi}{4}(4+2x^2)\right)\cdot\sec\left(\frac{\pi}{4}(2x^2+1)\right)$
$+ \pi x \csc\left(\frac{\pi}{4}(4+2x^2)\right)\cdot\sec\left(\frac{\pi}{4}(1+2x^2)\right)\tan\left(\frac{\pi}{4}(1+2x^2)\right)$

(a) $5\sqrt{2}\pi$

(b) $-5\sqrt{2}\pi$ Form(A) $f'(3) = 0 + (-3\sqrt{2}\pi) = -3\sqrt{2}\pi \Rightarrow$ Answ = (d)

(c) $3\sqrt{2}\pi$ Form(B) $f'(3) = 0 + (3\sqrt{2}\pi) = 3\sqrt{2}\pi \Rightarrow$ Answ = (c)

(d) $-3\sqrt{2}\pi$

Form(C) $f'(5) = 0 + (-5\sqrt{2}\pi) = -5\sqrt{2}\pi \Rightarrow$ Answ = (b)

(e) 0

2) Find the local linear approximation of

$$f(x) = \sqrt[4]{x} \quad at \quad x_0 = 16$$

$f'(x) = \frac{1}{4}x^{-3/4}$

$f'(16) = \frac{1}{4}(16)^{-3/4} = \frac{1}{32}$

then use it to approximate $\sqrt[4]{15} \approx$

$f(x) \cong \ell(x) = f'(16)(x-16) + f(16)$

(a) 1.9343

$= \frac{1}{32}(x-16) + \sqrt[4]{16}$

(b) 2.03054

$= \frac{x}{32} - \frac{1}{2} + 2 = \frac{x}{32} + \frac{3}{2}$

(c) 1.96799

Hence, $\ell(x) = \frac{x}{32} + \frac{3}{2}$ is the local linear approx.

(d) 2.03125

(e) 1.96875

Form (A) $\sqrt[4]{15} \cong \ell(15) = \frac{15}{32} + \frac{3}{2} = \frac{63}{32} = 1.96875 \Rightarrow$ (e)

Form (B) $\sqrt[4]{17} \cong \ell(17) = \frac{17}{32} + \frac{3}{2} = \frac{65}{32} = 2.03125 \Rightarrow$ (d)

Form (C) $\sqrt[4]{14} \cong \ell(14) = \frac{14}{32} + \frac{3}{2} = \frac{62}{32} = 1.9375 \Rightarrow$ (b)

3) Let $F(x) = f(3\sqrt{g(x)} - 2)$ where $f(x) = \frac{(x-1)(x^2+1)}{x^3+1}$ and $g(x) = f^{-1}(x)$.
Find $F'(0)$

$$f(x) = \frac{x^3 - x^2 + x - 1}{x^3+1} \Rightarrow f'(x) = \frac{(3x^2-2x+1)(x^3+1) - 3x^2(x^3-x^2+x-1)}{(x^3+1)^2}$$

$$f'(1) = \frac{(3-2+1)(2) - 0}{(1+1)^2} = \frac{4}{4} = 1 \Rightarrow \boxed{f'(1) = 1} \text{ — } (1)$$

$$\text{Let } z = g(0) = f^{-1}(0) \Leftrightarrow f(z) = 0 \Rightarrow \frac{(z-1)(z^2+1)}{z^3+1} = 0$$

$$\Rightarrow (z-1)(z^2+1) = 0 \Rightarrow z = 1 \Rightarrow \boxed{g(0) = 1} \text{ — } (2)$$

$$\text{Now, } g'(x) = \frac{1}{f'(f^{-1}(x))} = \frac{1}{f'(g(x))}$$

$$\text{So, } g'(0) = \frac{1}{f'(g(0))} = \frac{1}{f'(1)} \quad \text{use } (2)$$

use (1)

$$= \frac{1}{1} = 1$$

Hence $\boxed{g'(0) = 1}$ — (3)

$$\text{Now, } F'(x) = f'(3\sqrt{g(x)} - 2) \cdot \frac{d}{dx}\left[3\sqrt{g(x)} - 2\right]$$

$$= f'(3\sqrt{g(x)} - 2) \cdot 3 \cdot \frac{g'(x)}{2\sqrt{g(x)}}$$

$$\text{So, } F'(0) = f'(3\sqrt{g(0)} - 2) \cdot 3 \cdot \frac{g'(0)}{2\sqrt{g(0)}}$$

$$= f'(3\sqrt{1} - 2) \cdot 3 \cdot \frac{1}{2\sqrt{1}}$$

$$= f'(1) \cdot 3 \cdot \frac{1}{2} = (1)\cdot(3)\cdot\left(\frac{1}{2}\right) = \frac{3}{2}$$

Hence, $\boxed{F'(0) = \frac{3}{2}}$