

## SECTION 2.4

2.4.1 Find any points of discontinuity for  $f(x) = \frac{x-1}{x^2-1}$ .

2.4.2 Find any points of discontinuity for  $f(x) = \frac{x+1}{x^2+1}$ .

2.4.3 Show that  $f(x) = \frac{x^2-3}{x-\sqrt{3}}$  is not a continuous function.

2.4.4 Define  $f(x) = \frac{x^3+1}{x+1}$  so that it will be continuous everywhere.

2.4.5 Define  $g(x) = \frac{x^2+x-6}{x-2}$  so that it will be continuous everywhere.

2.4.6 Prove that  $f(x) = \sqrt{x^2+x}$  is continuous on  $[0, +\infty)$ .

2.4.7 Assign a value to the constant  $k$  which will make  $g$  continuous.

$$g(x) = \begin{cases} \frac{x+2}{x^3+2x^2+x+2}, & x \neq -2 \\ k, & x = -2 \end{cases}$$

2.4.8 Assign a value to the constant  $k$  which will make  $h$  continuous.

$$h(x) = \begin{cases} \frac{x^3+3x^2+x+3}{x+3}, & x \neq -3 \\ k, & x = -3 \end{cases}$$

2.4.9 Assign a value to the constant  $k$  which will make  $f$  continuous.

$$f(x) = \begin{cases} \frac{x^2-4x+3}{x-1}, & x \neq 1 \\ k, & x = 1 \end{cases}$$

2.4.10 Show that  $f(x) = \begin{cases} \frac{x^2-x-2}{x+1}, & x < -1 \\ 2x+2, & x \geq -1 \end{cases}$  is not continuous at  $x = -1$  but is continuous from the right at  $x = -1$ .

2.4.11 Examine  $h(x) = \begin{cases} \frac{2x^2+3x+1}{x+1}, & x < -1 \\ \frac{|x|}{x}, & -1 \leq x < 0 \\ 2x, & x \geq 0 \end{cases}$  and determine if  $h$  is (a) continuous at  $x = -1$ , (b) continuous at  $x = 0$ , and (c) continuous from the right at  $x = 0$ .

2.4.12 Examine  $g(x) = \begin{cases} \sqrt{\frac{2x+3}{2+x+x^2}}, & x < -1 \\ 2-x^2, & x \geq -1 \end{cases}$  and determine if  $g$  is (a) continuous at  $x = -1$ , (b) continuous from the right at  $x = -1$ , and (c) continuous from the left at  $x = -1$ .

$$2.4.13 \quad \text{Let } g(x) = \begin{cases} |x+1|, & x \leq -2 \\ x+1, & -2 < x < 1 \\ \sqrt{x+3}, & 1 \leq x \leq 6 \\ \frac{6}{8-x}, & 6 < x \leq 7 \\ 6, & 7 < x \leq 10 \end{cases} .$$

- (a) Determine if  $g$  is continuous from the right at  $x = -2$ .  
 (b) Determine if  $g$  is continuous from the left at  $x = 1$ .  
 (c) Determine if  $g$  is continuous at  $x = 7$ .  
 (d) Determine if  $g$  is continuous at  $x = 9$ .

2.4.14 Show that  $f(x) = \frac{x-1}{x(x+1)}$  is not continuous at  $x = 0$  or  $x = -1$  and show also that the discontinuities at  $x = 0$  and  $x = -1$  are nonremovable.

2.4.15 Show that  $f(x) = \frac{1}{(x-1)^3}$  is not continuous at  $x = 1$  and that the discontinuity at  $x = 1$  is nonremovable.

2.4.16 Show that the equation  $f(x) = x^3 + x + 6$  has at least one solution in the interval  $[-3, 0]$ .

2.4.17 Show that the equation  $f(x) = x^3 + 3x + 1$  has at least one solution in the interval  $[-1, 2]$ .

2.4.18 Determine the interval for which  $f(x) = \frac{1}{\sqrt{3-x}}$  is a continuous function.

2.4.19 Show that  $f(x) = \begin{cases} \frac{|x|}{x}, & x \neq 0 \\ k, & x = 0 \end{cases}$  cannot be made continuous for any assigned value of the constant  $k$ .