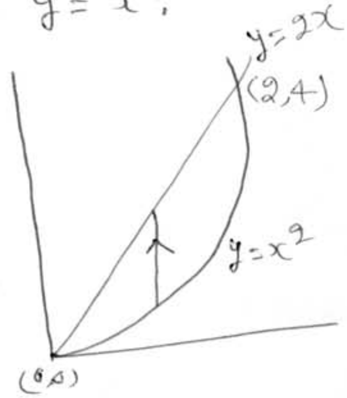
# QUIZ#6 ( Section 8 )

① Find the valume of the salid that lies under the paraboloid $z = x^2+y^2$ and above the region D in the xy-plane bounded by the line $y = 2x$ and the parabola $y = x^2$.




Sol. $D = \{(x,y) : 0 \le x \le 2,\ x^2 \le y \le 2x\}$

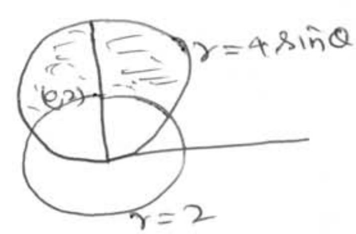
$$V = \iint_D (x^2+y^2)\,dA = \int_0^2 \int_{x^2}^{2x} (x^2+y^2)\,dy\,dx$$

$$= \int_0^2 \left[x^2 y + \frac{y^3}{3}\right]_{y=x^2}^{2x} dx = \int_0^2 \left(-\frac{x^6}{3} - x^4 + \frac{14x^3}{3}\right) dx$$

$$= \frac{216}{35}$$

Sol2 :- $V = \int_0^4 \int_{x=\frac{1}{2}y}^{x=\sqrt{y}} (x^2+y^2)\,dx\,dy = \cdots = \frac{216}{35}$

Q.2 Find the area of the region inside the circle $r = 4\sin\theta$ and outside the circle $r=2$




Sol $r=2$ and $r = 4\sin\theta \Rightarrow 2 = 4\sin\theta$

$\theta = \frac{\pi}{6}$ or $\frac{5\pi}{6}$

$$A = \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \int_2^{4\sin\theta} r\,dr\,d\theta = \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} [8\sin^2\theta - 2]\,d\theta$$

$$= \int [4(1-\cos 2\theta) - 2]\,d\theta = [2\theta - 2\sin 2\theta]_{\frac{\pi}{6}}^{\frac{5\pi}{6}} = \frac{4\pi}{3} + 2\sqrt{3}$$

Q.3:- Evaluate $\iint_D e^{x/y}\,dA$ , $D = \{(x,y) : 1 \le y \le 2,\ y \le x \le y^3\}$

Sol $\int_{y=1}^{2} \int_{x=y}^{y^3} e^{x/y}\,dx\,dy = \int_1^2 \left(y e^{\frac{x}{y}}\right)_y^{y^3} dy = \int_1^2 (y e^{y^2} - ye)\,dy$

$$= \left(\frac{1}{2}e^{y^2} - \frac{y^2}{2}e\right)_1^2$$

$$= \frac{1}{2}e^4 - 2e - \frac{1}{2}e + \frac{e}{2}$$

$$= \frac{1}{2}e^4 - 2e$$

# QUIZ #6 ( Section 12 )

1) Evaluate $\iint_D (x+2y)\, dA$ , $D = \{(x,y): -1 \le x \le 1,\ 2x^2 \le y \le 1+x^2\}$

Sol

$$\int_{-1}^{1}\int_{y=2x^2}^{1+x^2} (x+2y)\,dy\,dx = \int_{-1}^{1} (xy+y^2)\Big|_{y=2x^2}^{1+x^2} dx$$

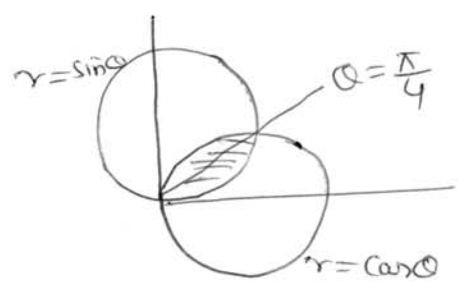
$$= \int_{-1}^{1} \left[x(1+x^2) + (1+x^2)^2 - x(2x^2) - (2x^2)^2\right] dx$$

$$= \int_{-1}^{1} (-3x^4 - x^3 + 2x^2 + x + 1)\, dx = \frac{32}{15}$$

2) Find the area of the region within both the circles $r = \cos\theta$ and $r = \sin\theta$.

Sol.

$$A = 2\int_0^{\pi/4}\int_0^{\sin\theta} r\,dr\,d\theta$$

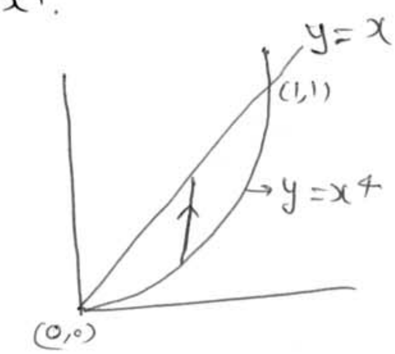



$$= \int_0^{\pi/4} \sin^2\theta\, d\theta = \int_0^{\frac{\pi}{4}} \left(\frac{1-\cos 2\theta}{2}\right) d\theta$$

$$= \frac{1}{2}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\pi/4} = \frac{\pi - 2}{8}$$

Q.3 :- Find the volume of the solid under the plane $x+2y-z=0$ and above the region bounded by $y = x$ and $y = x^4$.

Sol. $D = \{(x,y): 0 \le x \le 1,\ x^4 \le y \le x\}$




$$V = \iint_D (x+2y)\,dA$$

$$= \int_0^1 \int_{x^4}^{x} (x+2y)\,dy\,dx$$

$$= \int_0^1 \left[xy + y^2\right]_{y=x^4}^{x} dx = \int_0^1 (2x^2 - x^5 - x^8)\, dx$$

$$= \left(2\frac{x^3}{3} - \frac{x^6}{6} - \frac{x^9}{9}\right)\Big|_0^1 = \frac{2}{3} - \frac{1}{6} - \frac{1}{9} = \frac{12-3-2}{18} = \frac{7}{18}$$

Sol.2 :

$$V = \int_{y=0}^{1} \int_{x=y}^{x=y^{1/4}} (x+2y)\,dx\,dy$$