King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics
Math-201 Semester-082 QUIZ I

NAME: S.No. ID:

Maximum Marks: 10 Section: Time Allowed: 15 minutes

(1)For which value of t is the curve $x = 2sint, y = 3cost, \quad 0 < t < 2\pi$ concave upward.

(2) Find the length of the curve $x = e^t - t, y = 4e^{\frac{t}{2}}, \quad -8 \leq t \leq 3.$

Sol. 1 :- $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{-3\sin t}{2\cos t} = -\frac{3}{2}\tan t$

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{dx/dt} = \frac{-\frac{3}{2}\sec^2 t}{2\cos t} = -\frac{3}{4}\sec^3 t$$

The curve is concave upward $\frac{d^2y}{dx^2} \geq 0$

i.e. $\sec^3 t \leq 0 \Rightarrow \sec^3 t \leq 0$

$\sec^3 t < 0$

$\Rightarrow \frac{\pi}{2} < t < \frac{3\pi}{2}$

Sol. 2 :-

$$L = \int_{\alpha}^{\beta} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$$= \int_{-8}^{3} \sqrt{(e^t - 1)^2 + (2e^{t/2})^2}$$

$$= \int_{-8}^{3} \sqrt{e^{2t} + 1 - 2e^t + 4e^t}\, dt$$

$$= \int_{-8}^{3} \sqrt{(e^{2t} + 2e^t + 1)}\, dt = \int_{-8}^{3} \sqrt{(e^t + 1)^2}$$

$$= \int_{-8}^{3} (e^t + 1)\, dt = (e^t + t)_{-8}^{3} = e^3 + 3 - e^{-8} + 8$$

$$= e^3 + 11 - e^{-8}$$

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(1)Find an equation of the tangent line to the parametric curve given by $x = e^t, y = (t-1)^2$ at (1,1).

(2)Find the area of the surface generated by revolving the curve $x = a\cos^3\theta, y = a\sin^3\theta, \ 0 \le \theta \le \frac{\pi}{2}$. about X-axis.

Sol.1 :- $\frac{dy}{dt} = 2(t-1)$, $\frac{dx}{dt} = e^t$; $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2(t-1)}{e^t}$

At (1,1), $t = 0$. Therefore $\frac{dy}{dx} = -2$.

The equation of Tangent is $y - 1 = -2(x-1)$ or $\boxed{y = -2x+3}$

Second Solution :- $x = e^t \Rightarrow \ln x = t$

$y = (t-1)^2 = (\ln x - 1)^2$

$\frac{dy}{dx} = 2(\ln x - 1)(\frac{1}{x})$

$\frac{dy}{dx}\Big|_{(1,1)} = 2(-1)(1) = -2$

Sal.2 :- $\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2 = (-3a\cos^2\theta\sin\theta)^2 + (3a\sin^2\theta\cos\theta)^2$

$= 9a^2\sin^2\theta\cos^2\theta$

$$S = 2\pi a \int_0^{\pi/2} \sin^3\theta \sqrt{9a^2\sin^2\theta\cos^2\theta}\, d\theta$$

$$= 6\pi a^2 \int_0^{\pi/2} \sin^4\theta\cos\theta\, d\theta = \frac{6}{5}\pi a^2 \left[\sin^5\theta\right]_0^{\pi/2}$$

$$= \frac{6}{5}\pi a^2.$$