King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics
Math-201 Semester-091 QUIZ I

NAME: S.No. ID:

Maximum Marks: 10 Section:04 Time Allowed: 15 minutes

(1) If $x = 2sint, y = 3cost, 0 < t < 2\pi$, find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.For which values of t is the curve concave upward ?

(2)Find the length of the curve $x = e^t + e^{-t}, y = 5 - 2t, 0 \le t \le 3$.

Sol.1 :- $\frac{dy}{dx} = \frac{-3\sin t}{2\cos t} = -\frac{3}{2}\tan t$

$$\frac{d^2y}{dx^2} = \frac{-\frac{3}{2}\sec^2 t}{2\cos t} = -\frac{3}{4}\sec^3 t$$

Concave upward $\frac{d^2y}{dx^2} > 0$ i.e $\sec^3 t < 0 \Rightarrow \sec t < 0$

$$\Longrightarrow \frac{\pi}{2} < t < \frac{3\pi}{2}$$

Sol 2 :- $\frac{dx}{dt} = e^t - e^{-t}$; $\frac{dy}{dt} = -2$

$$L = \int ds = \int_0^3 \frac{ds}{dt}\, dt$$

$$= \int_0^3 \sqrt{e^{2t} + e^{-2t} - 2 + 4}\; dt = \int_0^3 \sqrt{(e^t + e^{-t})^2}\, dt$$

$$= \int_0^3 (e^t + e^{-t})\, dt = (e^t - e^{-t})\Big|_0^3$$

$$= e^3 - e^{-3}$$

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(1) Convert parametric equations $x = 2cott\ sec^2t$ and $y = cosect$ in cartesian equation.

(2)Find the surface area by rotating the curve $x = 3t - t^3, y = 3t^2, 0 \le t \le 1$ about x-axis.

Sol. 1 :- $x = 2\frac{\cos t}{\sin t} \cdot \frac{1}{\cos^2 t} = \frac{2}{\sin t \cos t} = 2 \operatorname{cosec} t \frac{1}{\cos t}$ —①

and $y = \frac{1}{\sin t}$ or $\sin t = \frac{1}{y}$

$\cos t = \sqrt{1 - \frac{1}{y^2}} = \sqrt{\frac{y^2-1}{y^2}} = \frac{\sqrt{y^2-1}}{y}$ —②

① & ② $\Rightarrow x = 2y \frac{1}{\frac{\sqrt{y^2-1}}{y}} = \frac{2y^2}{\sqrt{y^2-1}}$

Sol. 2 : $\frac{ds}{dt} = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = \sqrt{(3-3t^2)^2 + (6t)^2}$

$= \sqrt{18t^2 + 9t^4 + 9} = \sqrt{9(1+t^2)^2}$

$= 3(1+t^2)$

$S = 2\pi \int y\, ds = 2\pi \int_0^1 3t^2 \cdot 3(1+t^2)\, dt$

$= 18\pi \int_0^1 (t^2 + t^4)\, dt = 18\pi \left[\frac{t^3}{3} + \frac{t^5}{5}\right]_0^1 = \frac{48}{5}\pi$

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(1) Find the points on the curve $x = 2t^3 + 3t^2 - 12t$, $y = 2t^3 + 3t^2 + 1$, where the tangent is horizontal or vertical.

(2) If $x = t^3 - 12t$, $y = t^2 - 1$, find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. For which values of t is the curve concave upward.

Sol. 1: $\frac{dy}{dt} = 0 \Rightarrow 6t^2 + 6t = 0 \Rightarrow 6t(t+1) = 0 \Rightarrow t = 0$ or -1

Points $(x, y) = (0, 1)$ or $(13, 2)$

The curve has horizontal tangents at $(0, 1)$ and $(13, 2)$.

$$\frac{dx}{dt} = 0 \Rightarrow 6t^2 + 6t - 12 = 0$$
$$\Rightarrow 6(t-1)(t+2) = 0 \Rightarrow t = 1 \text{ or } -2$$

$(x, y) = (-7, 6)$ or $(20, -3)$

The curve has vertical tangents at $(-7, 6)$ and $(20, -3)$.

Sol. 2 :- $\frac{dx}{dt} = 3t^2 - 12$; $\frac{dy}{dt} = 2t$

$$\frac{dy}{dx} = \frac{2t}{3t^2 - 12}$$

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{2t}{3t^2-12}\right)}{\frac{dx}{dt}} = \frac{\frac{2(3t^2-12) - 2t \cdot 6t}{(3t^2-12)^2}}{(3t^2-12)}$$

$$= -\frac{6t^2 - 24}{(3t^2-12)^3} = \frac{-6(t^2+4)}{27(t^2-4)^3} = \frac{-2(t^2+4)}{9(t^2-4)^3}$$

The curve is concave upward

$$t^2 - 4 < 0$$
$$\Rightarrow |t| < 2$$
$$\Rightarrow \boxed{-2 < t < 2}$$