King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics
Math-201 Semester-083 QUIZ I

NAME: S.No. ID:

Maximum Marks: 10 Section:02 Time Allowed: 15 minutes

(1)Find cartesian equation of the curve whose polar equation is given as $r = tan\theta sin\theta$. (4)

(2)Sketch the curve $r = 1 + sin\theta$ and find the slope of tangent line when $\theta = \frac{\pi}{3}$. (6)

$x = r\cos\theta, y = r\sin\theta$

$\tan\theta = \frac{y}{x}$

Sol.1 : $r = \tan\theta \sin\theta$

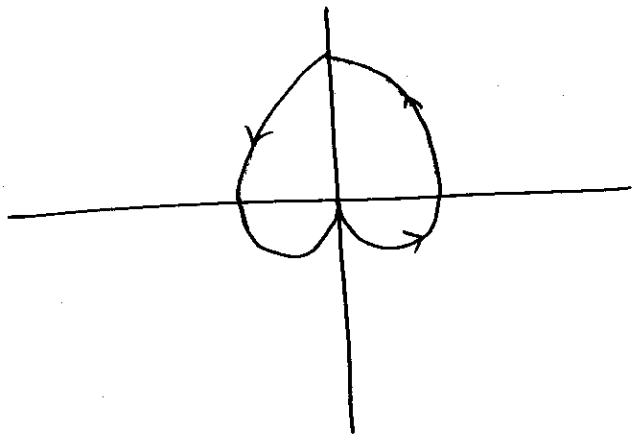
$= \frac{\sin\theta}{\cos\theta} \cdot \sin\theta = \frac{y}{x} \cdot \frac{y}{r}$

$r^2 = \frac{y^2}{x}$

$x^2 + y^2 = \frac{y^2}{x}$ Ans

Sol. 2 :- $r = 1 + \sin\theta$

θ	r
0	1
$\frac{\pi}{2}$	2
π	1
$\frac{3\pi}{2}$	0
2π	1



Parametric equations are

$x = r\cos\theta = (1+\sin\theta)\cos\theta = \cos\theta + \sin\theta\cos\theta$

$y = r\sin\theta = (1+\sin\theta)\sin\theta = \sin\theta + \sin^2\theta$

$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} =$ ~~$\frac{\sin\theta + \cos^2\theta - \sin^2\theta}{\quad}$~~

$= \frac{\cos\theta + 2\sin\theta\cos\theta}{-\sin\theta + \cos^2\theta - \sin^2\theta}$

$\left.\frac{dy}{dx}\right|_{\theta=\frac{\pi}{3}} = \frac{\cos\frac{\pi}{3} + 2\sin\frac{\pi}{3}\cos\pi/3}{-\sin\frac{\pi}{3} + \cos^2\frac{\pi}{3} - \sin^2\frac{\pi}{3}} = \frac{\frac{1}{2} + 2\times\frac{1}{2}\frac{\sqrt{3}}{2}}{-\frac{\sqrt{3}}{2} + \frac{1}{4} - \frac{3}{4}}$

$= \frac{(1+\sqrt{3})/2}{-\frac{\sqrt{3}}{2} - \frac{1}{2}} = \frac{(1+\sqrt{3})/2}{-(1+\sqrt{3})/2} = -1$

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Maximum Marks: 10 Section:06 Time Allowed: 15 minutes

(1)Find cartesian equation of the curve whose polar equation is given as $r = -cot\theta sec\theta$. 04

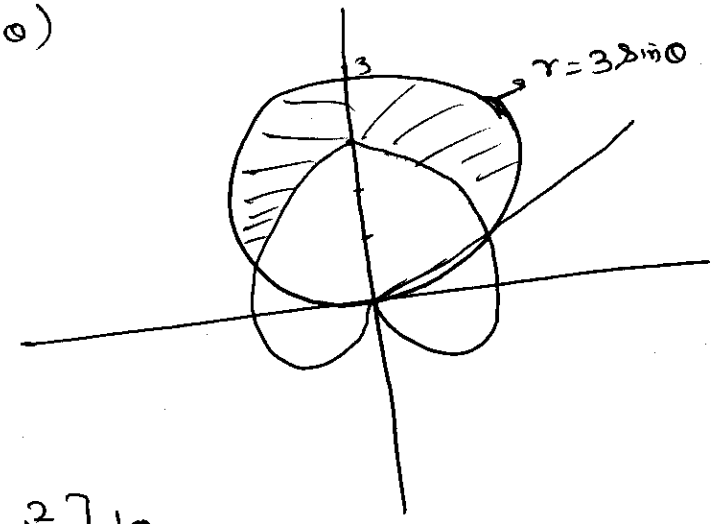
(2)Find the area of the region that lies inside the curve $r = 3sin\theta$ and outside the curve $r = 1 + sin\theta$.. 06

Sol I: $r = -\cot\theta\sec\theta = -\frac{\cos\theta}{\sin\theta}\cdot\frac{1}{\cos\theta}$

$r = -\frac{1}{\sin\theta}$

$r = -\frac{r}{y}$ $(y = r\sin\theta)$

$\Rightarrow \boxed{y = -1}$



Sol 2: $r = 3\sin\theta = 1+\sin\theta$

$\Rightarrow 2\sin\theta = 1$

$\Rightarrow \sin\theta = \frac{1}{2}$

$\Rightarrow \theta = \frac{\pi}{6}$ or $\frac{5\pi}{6}$

$$A = 2\times\frac{1}{2}\int_{\frac{\pi}{6}}^{\pi/2}\left[9\sin^2\theta - (1+\sin\theta)^2\right]d\theta$$

$$= \int_{\frac{\pi}{6}}^{\pi/2}\left[9\sin^2\theta - 1 - \sin^2\theta - 2\sin\theta\right]d\theta = \int_{\frac{\pi}{6}}^{\pi/2}\left[8\sin^2\theta - 1 - 2\sin\theta\right]d\theta$$

$$= \int_{\frac{\pi}{6}}^{\pi/2} 8\left(\frac{1-\cos 2\theta}{2}\right) - 1 - 2\sin\theta\, d\theta$$

$$= \int_{\frac{\pi}{6}}^{\pi/2}\left[-4\cos 2\theta + 3 - 2\sin\theta\right]d\theta = \left[3\theta - 2\sin 2\theta + 2\cos\theta\right]_{\pi/6}^{\pi/2}$$

$$= \pi \quad \underline{\text{Ans}}$$