King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics
Math-201 Semester-092 QUIZ II

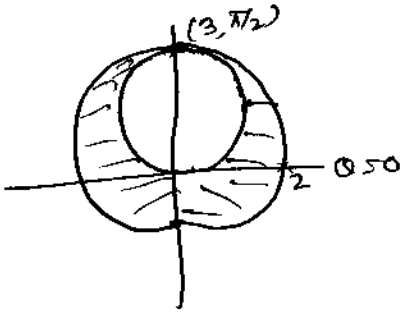
NAME: S.No. ID:

Maximum Marks: 15 Section: Time Allowed: 25 minutes

(1)Find the area of the region that lies inside the curve $r = 2 + 2sin\theta$ and outside the curve $r = 3sin\theta$.

(2)Find the exact length of the polar curve $r = e^{2\theta}$, $0 \le \theta \le 2\pi$.

(3)Find the vector projection of $< 1, 1, 2 >$ onto $< -2, 3, 1 >$.



Sol. 1 The area inside the curve $r = 2 + 2\sin\theta$ is

$$\int_0^{2\pi} \frac{1}{2}(2+2\sin\theta)^2 \, d\theta$$

$$= \frac{1}{2}\int_0^{2\pi} (4 + 4\sin\theta + 2\sin^2\theta)\, d\theta$$

$$= \frac{1}{2}\int_0^{2\pi} \left(4 + 4\sin\theta + \frac{1-\cos 2\theta}{2}\right) d\theta$$

$$= \frac{1}{2}\int_0^{2\pi} \left(\frac{9}{2} + 4\sin\theta - \frac{1}{2}\cos 2\theta\right) d\theta$$

$$= \frac{1}{2}\left[\frac{9}{2}\theta - 4\cos\theta - \frac{1}{4}\sin 2\theta\right]_0^{2\pi} = \frac{9\pi}{2}$$

Shaded area $= \frac{9\pi}{2} - \pi\left(\frac{3}{2}\right)^2 = \frac{9\pi}{2} - \frac{9\pi}{4} = \frac{9\pi}{4}$

Sol 2 $L = \int_0^{2\pi} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} = \int_0^{2\pi} \sqrt{(e^{2\theta})^2 + (2e^{2\theta})^2}\, d\theta = \int_0^{2\pi} \sqrt{5e^{4\theta}}\, d\theta$

$$= \sqrt{5}\int_0^{2\pi} e^{2\theta}\, d\theta = \frac{\sqrt{5}}{2}\left(e^{4\pi} - 1\right)$$

Sol. 3: $\vec{b} = \langle 1, 1, 2\rangle$, $\vec{a} = \langle -2, 3, 1\rangle$

$$\text{Proj}_{\vec{a}}\vec{b} = \frac{(\vec{a}\cdot\vec{b})\,\vec{a}}{|\vec{a}|^2} = \frac{(-2+3+2)\langle -2, 3, 1\rangle}{(\sqrt{14})^2}$$

$$= \frac{3}{14}\langle -2, 3, 1\rangle = \left\langle -\frac{3}{7}, \frac{9}{14}, \frac{1}{14}\right\rangle$$