King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics

Math 201 - Calculus III
FINAL EXAM
Term 082
Monday, 22 June 2009
Time Allowed: 180 minutes

MASTER **MASTER**

Name :
ID :
Section :

Check that this exam has 24 questions

Important Instructions:

1. All types of calculators, pagers or mobile phones are NOT allowed during the examination.
2. Use HB 2.5 pencils only.
3. Use a good eraser. DO NOT use the eraser attached to the pencil.
4. Write your name, ID number and Section number on the examination paper and in the upper left corner of the answer sheet.
5. When bubbling your ID number and Section number, be sure that the bubbles match with the numbers that you write.
6. The Test Code Number is already bubbled in your answer sheet. Make sure that it is the same as that printed on your question paper.
7. When bubbling, make sure that the bubbled space is fully covered.
8. When erasing a bubble, make sure that you do not leave any trace of penciling.

1. The positions of two particles P_1 and P_2 at time t $(0 \le t \le 2\pi)$ are given by:

$$P_1 : x_1 = 3\sin t, \quad y_1 = 2\cos t$$

$$P_2 : x_2 = -3 + \cos t, \quad y_2 = 1 + \sin t.$$

These particles meet at the point

(A) $(-3, 0)$

(B) $(0, 2)$

(C) $(-1, 2)$

(D) $(0, \frac{1}{2})$

(E) $(-1, -3)$

2. The polar equation $r = \dfrac{6}{3\cos\theta + 2\sin\theta}$ represents a

(A) line

(B) circle

(C) hyperbola

(D) parabola

(E) cardioid

3. If the angle between two unit vectors $\vec{u}_1$ and $\vec{u}_2$ is $\frac{\pi}{3}$, then $|\,2\vec{u}_1 - \vec{u}_2|$ is equal to

(A) $\sqrt{3}$

(B) 3

(C) 1

(D) $5 - \sqrt{3}$

(E) $3 - \sqrt{3}$

4. A parallelepiped is determined by the vectors $\vec{u} = \langle 0, 4, 2\rangle$, $\vec{v} = \langle 0, 4, -1\rangle$, $\vec{w} = \langle m, 1, 3\rangle$, where m is a positive real number. If the volume of the parallelepiped is 60, then m is equal to

(A) 5

(B) 10

(C) 6

(D) 12

(E) 4

5. The plane through the point $(4, -2, 3)$ and parallel to the plane $x+3y-2z = 12$ contains the point

   (A) $(-3, -1, 1)$

   (B) $(3, -1, 0)$

   (C) $(2, 1, 1)$

   (D) $(-2, -3, 1)$

   (E) $(-1, -2, 1)$

6. The equation $x^2 - y^2 + z^2 - 2x + 2y + 4z + 2 = 0$ represents a

   (A) hyperboloid of one sheet with center $(1, 1, -2)$

   (B) sphere with center $(1, 1, -2)$

   (C) hyperboloid of two sheets with center $(1, 1, -2)$

   (D) circular cone with vertex $(1, 1, -2)$

   (E) paraboloid with vertex $(1, 1, -2)$

7. Let $L = \lim_{(x,y)\to(0,0)} \dfrac{\sin(\sqrt{x^2+y^2})}{\sqrt{2x^2+2y^2}}$. Then

   (A) $L = \dfrac{\sqrt{2}}{2}$

   (B) $L = \sqrt{2}$

   (C) $L = 0$

   (D) L does not exist

   (E) $L = 1$

8. Let $f(x,t) = x^2 e^{-t/2}$. The partial derivative $f_{txx}(x,t)$ is

   (A) $-e^{-t/2}$

   (B) $e^{-t/2}$

   (C) $xe^{-t/2}$

   (D) $-xe^{-t/2}$

   (E) e^{-t}

9. Using the linear approximation of the function $f(x, y) = \sqrt{20 - x^2 - 7y^2}$ at $(2, 1)$, the value of $f(1.97, 1.06)$ is approximately equal to

(A) 2.88

(B) 2.98

(C) 3.08

(D) 3.16

(E) 3.06

10. Let $z = \tan^{-1}(u/v)$, where $u(x, y) = 2x + y$ and $v(x, y) = 3x - y$. Then $\partial z/\partial y$ at the point $(x, y) = (1, 1)$ is equal to

(A) $\frac{5}{13}$

(B) 0

(C) $\frac{3}{13}$

(D) $\frac{1}{13}$

(E) $\frac{7}{13}$

11. The points on the surface $x^2 + 2y^2 + 3z^2 = 12$ at which the tangent plane is perpendicular to the line with parametric equations $x = 1 + 2t$, $y = 3 + 8t$, $z = 2 - 6t$ are

(A) $(1, 2, -1)$ and $(-1, -2, 1)$

(B) $(-1, 2, 1)$ and $(1, -2, -1)$

(C) $(-1, -2, -1)$ and $(1, 2, 1)$

(D) $(2, -2, 0)$ and $(-2, 2, 0)$

(E) $(2, 2, 0)$ and $(-2, -2, 0)$

12. If $f(x, y) = -y^3 + 4xy - 2x^2 + 1$, then f has

(A) a local maximum at $(\frac{4}{3}, \frac{4}{3})$ and a saddle point at $(0, 0)$

(B) a local maximum at $(0, 0)$ and a local minimum at $(\frac{4}{3}, \frac{4}{3})$

(C) a local maximum at $(0, 0)$ and a saddle point at $(\frac{4}{3}, \frac{4}{3})$

(D) a local maximum at $(0, 0)$ and $(\frac{4}{3}, \frac{4}{3})$

(E) a local minimum at $(\frac{4}{3}, \frac{4}{3})$ and a saddle point at $(0, 0)$

13. The minimum value of the function $f(x, y, z) = 2x + 6y + 10z$ subject to the constraint $x^2 + y^2 + z^2 = 35$ is

(A) -70

(B) -60

(C) 0

(D) 12

(E) -80

14. Using a Riemann sum with $m = n = 2$ and lower left corners as the sample points, the approximate value of the double integral

$$\iint_R (x + 2y)dA,$$

where $R = [0, 1] \times [0, 1]$ is

(A) $\frac{3}{4}$

(B) 1

(C) -2

(D) $-\frac{2}{3}$

(E) $\frac{3}{2}$

15. The volume of the solid bounded by the surface $z = 2x\sqrt{x^2 + y}$ and the planes $x = 0$, $x = 1$, $y = 0$, $y = 3$ and $z = 0$ is

(A) $\frac{4}{15}(31 - 9\sqrt{3})$

(B) $\frac{4}{15}(32 - 9\sqrt{3})$

(C) $\frac{2}{15}(32 - 9\sqrt{3})$

(D) $\frac{2}{15}(31 - 9\sqrt{3})$

(E) $\frac{1}{15}(31 - 9\sqrt{3})$

16. Let $R = \{(x, y)|\ 0 \leq x \leq \pi,\quad 0 \leq y \leq \pi/2\}$. Then

$$\iint_R \cos(x + 2y)dA$$

is equal to

(A) -2

(B) -4

(C) 0

(D) π

(E) $\pi/2$

17. The value of the iterated integral

$$\int_0^1 \int_{\sin^{-1} y}^{\pi/2} \cos x \sqrt{1+\cos^2 x}\ dxdy$$

is equal to

(A) $\dfrac{2\sqrt{2}-1}{3}$

(B) $\dfrac{\pi}{3}$

(C) $\pi - 2\sqrt{2}$

(D) $\dfrac{2\sqrt{2}}{3}$

(E) $\dfrac{2}{3}$

18. The value of the double integral

$$\iint_D e^{x/y}\ dA,$$

where $D = \{(x, y)|1 \le y \le 2,\ y \le x \le y^3\}$, is equal to

(A) $\dfrac{e^4 - 4e}{2}$

(B) $\dfrac{e^3 - 3}{2}$

(C) $\dfrac{e^4}{2}$

(D) $\dfrac{1}{4}$

(E) $\dfrac{e^2 - 2e}{4}$

19. The value of the double integral

$$\iint\limits_R \sqrt{4 - x^2 - y^2}\, dA,$$

where $R = \{(x, y)|\ x^2 + y^2 \leq 4,\ x \geq 0\}$, is equal to

(A) $\dfrac{8\pi}{3}$

(B) $\dfrac{8\pi}{5}$

(C) $\dfrac{8\pi}{7}$

(D) π

(E) 2π

20. The value of the double integral

$$\iint\limits_R \tan^{-1}\left(\frac{y}{x}\right)\, dA,$$

where $R = \left\{(x, y)|1 \leq x^2 + y^2 \leq 4, 0 \leq y \leq \sqrt{3}\, x\right\}$, is equal to

(A) $\dfrac{\pi^2}{12}$

(B) $\dfrac{\pi^2}{18}$

(C) $\dfrac{3\pi^2}{8}$

(D) $\dfrac{\pi}{12}$

(E) $\dfrac{3\pi}{16}$

21. Let E be a solid bounded by the planes $z = 0$, $x = 0$, $y = 1$, and $z = y - x$. The expression of

$$\iiint_E f(x, y, z)\, dV$$

as an iterated integral is

(A) $\int_0^1 \int_0^y \int_0^{y-x} f(x, y, z)\, dzdxdy$

(B) $\int_0^1 \int_x^2 \int_0^{y-x} f(x, y, z)\, dzdydx$

(C) $\int_0^1 \int_0^x \int_0^y f(x, y, z)\, dxdydz$

(D) $\int_0^1 \int_0^y \int_1^2 f(x, y, z)\, dydxdz$

(E) $\int_0^2 \int_x^1 \int_0^{y-x} f(x, y, z)\, dzdydx$

22. The volume of the solid bounded by the cone $z = \sqrt{x^2 + y^2}$, the cylinder $x^2 + y^2 = 9$ and the xy-plane is

(A) 18π

(B) 9π

(C) 2π

(D) π

(E) 12π

23. The value of the iterated integral

$$\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{0}^{\sqrt{4-x^2-y^2}} z^2\sqrt{x^2+y^2+z^2}\ \ dzdydx$$

is equal to

(A) $\frac{64\pi}{9}$

(B) -2π

(C) 3π

(D) $1-\frac{1}{e}$

(E) 0

24. The solid S is enclosed by the hemispheres $y=\sqrt{9-x^2-z^2}$, $y=\sqrt{4-x^2-z^2}$, and the plane $y=0$. The y-coordinate $\bar{y}$ of the centroid of S is

$$\left(\text{Hint: } \bar{y}=\frac{\iiint\limits_S y\ dV}{\text{Volume of } S}\right)$$

(A) $\frac{195}{152}$

(B) 0

(C) $\frac{152}{195}$

(D) $\frac{175}{152}$

(E) $\frac{19}{15}$