King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics

Math 201 - Calculus III
FINAL EXAM (Term 083)

Thursday, 3 September 2009

Time Allowed: **150** minutes **MASTER**

Name :

ID :

Section :

Check that this exam has 20 questions

Important Instructions:

1. *All types of calculators, pagers or mobile phones are NOT allowed during the examination.*
2. *Use HB* 2.5 *pencils only.*
3. *Use a good eraser. DO NOT use the eraser attached to the pencil.*
4. *Write your name, ID number and Section number on the examination paper and in the upper left corner of the answer sheet.*
5. *When bubbling your ID number and Section number, be sure that the bubbles match with the numbers that you write.*
6. *The Test Code Number is already bubbled in your answer sheet. Make sure that it is the same as that printed on your question paper.*
7. *When bubbling, make sure that the bubbled space is fully covered.*
8. *When erasing a bubble, make sure that you do not leave any trace of penciling.*

1. The curve $C: \ x = t^3 - 12t, \ y = t^2$ is concave upward on the interval

   (A) $-2 < t < 2$

   (B) $t > 2$

   (C) $t < -2$

   (D) $-2 \leq t \leq 2$

   (E) $t < 0$

2. The length of the curve $x = e^t + e^{-t}, \ y = 5 - 2t, \quad 0 \leq t \leq 3$ is equal to

   (A) $e^3 - e^{-3}$

   (B) $e^3 + e^{-3}$

   (C) $e^3 - e^{-3} - 2$

   (D) $e^3 + e^{-3} + 2$

   (E) $e^3 + e^{-3} - 2$

3. The area of the region that lies inside the circle $r = 3\sin\theta$ and outside the cardioid $r = 1 + \sin\theta$ is

   (A) π

   (B) 2π

   (C) $\frac{\pi}{2}$

   (D) $\pi - 2$

   (E) $\pi + 2$

4. The vector projection of $\vec{b} = \langle 1, 1, 2\rangle$ onto $\vec{a} = \langle -1, 1, 3\rangle$ is equal to

   (A) $\left\langle -\frac{6}{11}, \frac{6}{11}, \frac{18}{11} \right\rangle$

   (B) $\left\langle -\frac{6}{11}, \frac{1}{11}, \frac{3}{11} \right\rangle$

   (C) $\left\langle -\frac{6}{\sqrt{11}}, \frac{8}{\sqrt{11}}, \frac{18}{\sqrt{11}} \right\rangle$

   (D) $\left\langle \frac{8}{\sqrt{11}}, \frac{1}{\sqrt{11}}, \frac{3}{\sqrt{11}} \right\rangle$

   (E) $\left\langle -\frac{6}{\sqrt{11}}, \frac{1}{\sqrt{11}}, \frac{3}{\sqrt{11}} \right\rangle$

5. A vector perpendicular to the plane that passes through the points $P(1, 4, 6)$, $Q(-2, 5, -1)$, $R(1, -1, 1)$ is

   (A) $\langle -8, -3, 3 \rangle$

   (B) $\langle 6, -3, 3 \rangle$

   (C) $\langle -8, -3, -3 \rangle$

   (D) $\langle -8, 3, 3 \rangle$

   (E) $\langle -6, 3, 3 \rangle$

6. The intersection point between the line $x = y - 1 = 2z$ and the plane $4x - y + 3z = 8$ is

   (A) $(2, 3, 1)$

   (B) $(1, 2, 1/2)$

   (C) $(3, 2, 1)$

   (D) $(-2, -1, -1)$

   (E) $(-2, -3, 1)$

7. If

$$f(x,y) = \begin{cases} \dfrac{x^2y^3}{2x^2+y^2}, & (x,y) \neq (0,0) \\ 1, & (x,y) = (0,0), \end{cases}$$

then

(A) f is not continuous at $(0,0)$

(B) $\lim_{(x,y)\to(0,0)} f(x,y) = 1$

(C) $f_x(0,0) = 1$

(D) f is continuous everywhere

(E) $\lim_{(x,y)\to(0,0)} f(x,y)$ does not exist

8. Let $f(x,t) = \tan^{-1}(x\sqrt{t})$. The value of $f_{xt}(2,1)$ is

(A) $-\dfrac{3}{50}$

(B) 0

(C) $\dfrac{3}{50}$

(D) $\dfrac{1}{10}$

(E) $-\dfrac{1}{10}$

9. If $z = e^y \sin^{-1} x$, $x = \cos(u+v)$, $y = u \ln v$, then $\dfrac{\partial z}{\partial v} - \dfrac{\partial z}{\partial u}$ is equal to

   (A) $\dfrac{z}{v}(u - v \ln v)$

   (B) $\dfrac{y}{v}(u - v \ln v)$

   (C) $\dfrac{x}{v}(z - v \ln v)$

   (D) $\dfrac{z}{u}(v - u \ln v)$

   (E) $\dfrac{z}{x}(u - y \ln v)$

10. Using linear approximation of the function $f(x, y, z) = \sqrt{x^2 + y^2 + z^2}$ at $(3, 2, 6)$, the value of $\sqrt{(3.02)^2 + (1.97)^2 + (5.93)^2}$ is approximately equal to

    (A) 6.94

    (B) 6.96

    (C) 7.04

    (D) 7.05

    (E) 7.06

11. The points on the hyperboloid $x^2 - y^2 + 2z^2 = 2$ at which the normal line is parallel to the line that contains the points $(3, -1, 0)$ and $(4, 0, -2)$ are

    (A) $(1, -1, -1)$ and $(-1, 1, 1)$

    (B) $(\sqrt{2}, 0, -1)$ and $(-\sqrt{2}, 0, 1)$

    (C) $(-1, -1, 1)$ and $(1, 1, -1)$

    (D) $(1, -1, 1)$ and $(-1, 1, -1)$

    (E) $(\sqrt{2}, 0, 1)$ and $(-\sqrt{2}, 0, -1)$

12. Let $f(x, y) = 2x^2 - 4x + y^2 - 2y + 1$ and D be the closed triangular region bounded by $x = 0$, $y = 2$ and $y = 2x$ in the first quadrant. The absolute minimum value of f is equal to

    (A) $-\frac{5}{3}$

    (B) -2

    (C) -1

    (D) 0

    (E) $-\frac{4}{3}$

13. Let P and Q be two points on the sphere $x^2 + y^2 + z^2 = 4a^2$ that are closest to and farthest from $M(3, 1, -1)$, respectively. Then P and Q are given by

(A) $P\left(\frac{6a}{\sqrt{11}}, \frac{2a}{\sqrt{11}}, -\frac{2a}{\sqrt{11}}\right)$ and $Q\left(-\frac{6a}{\sqrt{11}}, -\frac{2a}{\sqrt{11}}, \frac{2a}{\sqrt{11}}\right)$

(B) $P\left(\frac{6a}{11}, \frac{2a}{11}, -\frac{2a}{11}\right)$ and $Q\left(-\frac{6a}{11}, -\frac{2a}{11}, \frac{2a}{11}\right)$

(C) $P\left(-\frac{6a}{\sqrt{11}}, -\frac{2a}{\sqrt{11}}, -\frac{2a}{\sqrt{11}}\right)$ and $Q\left(\frac{6a}{\sqrt{11}}, \frac{2a}{\sqrt{11}}, \frac{2a}{\sqrt{11}}\right)$

(D) $P\left(-\frac{6a}{11}, -\frac{2a}{11}, -\frac{2a}{11}\right)$ and $Q\left(\frac{6a}{11}, \frac{2a}{11}, \frac{2a}{11}\right)$

(E) $P\left(\frac{2a}{\sqrt{11}}, \frac{2a}{\sqrt{11}}, -\frac{6a}{\sqrt{11}}\right)$ and $Q\left(-\frac{2a}{\sqrt{11}}, -\frac{2a}{\sqrt{11}}, \frac{6a}{\sqrt{11}}\right)$

14. The volume of the solid bounded by the surface $\rho = a$ is given by

(A) $\int_0^{2\pi} \int_0^{\pi} \int_0^{a} \rho^2 \sin\phi \; d\rho d\phi d\theta$

(B) $\int_0^{2\pi} \int_0^{2\pi} \int_0^{a} \rho^2 \sin\phi \; d\rho d\phi d\theta$

(C) $\int_0^{2\pi} \int_0^{\pi} \int_0^{a} d\rho d\phi d\theta$

(D) $\int_0^{\pi} \int_0^{\pi} \int_{-a}^{a} d\rho d\phi d\theta$

(E) $\int_0^{2\pi} \int_0^{\pi} \int_{-a}^{a} \rho^2 \sin\phi \; d\rho d\phi d\theta$

15. The volume of the solid bounded by $x+y+z=3$ and the planes $x=0$, $x=a$, $y=0$, $y=b$ and $z=0$ is

    (A) $\frac{ab}{2}(6-a-b)$

    (B) $\frac{ab}{2}(a^2-b^2)$

    (C) $ab(a^2+b^2)$

    (D) a^3+b^3

    (E) $ab(a^3+b^3)$

16. If $R=\{(x,y)|\ 0\le x\le a,\quad 0\le y\le b\}$,

    then $\displaystyle\iint_R (xy^4+yx^4)\ dA$ is equal to

    (A) $\frac{a^2b^2}{10}(a^3+b^3)$

    (B) $\frac{a^3b^3}{10}(a^2+b^2)$

    (C) $\frac{a^2b^2}{10}(a^3-b^3)$

    (D) $\frac{a^2b^2}{10}(b^3-a^3)$

    (E) $\frac{a^2b^2}{10}$

17. The volume of the solid bounded by $z = a$ and $z = a\sqrt{x^2 + y^2}$ $(a > 0)$ is

(A) $\dfrac{\pi a}{3}$

(B) $\dfrac{4\pi a}{3}$

(C) $\dfrac{\pi a^2}{3}$

(D) $\dfrac{\pi a^3}{3}$

(E) $\dfrac{2\pi a^3}{3}$

18. Let E be the solid outside the cylinder $x^2 + y^2 = 1,\ 0 \le z \le 1$ and inside the hemisphere $x^2 + y^2 + z^2 = 4,\ z \ge 0$. The volume of E is equal to

(A) $2\sqrt{3}\pi$

(B) 2π

(C) $\sqrt{3}\pi$

(D) 4π

(E) π

19. Let E be the solid bounded by $z = 0$, $z = x^2 + y^2$, $y = x^2$ and $x = y^2$. Then the volume of E is equal to

(A) $\frac{6}{35}$

(B) $\frac{3}{14}$

(C) $\frac{5}{4}$

(D) $\frac{7}{35}$

(E) $\frac{1}{5}$

20. Let D be the region bounded by the line $y = x-1$ and the parabola $y^2 = 2x+6$.

The value of the double integral $\iint_D \sqrt{3}xy \, dA$ is equal to

(A) $36\sqrt{3}$

(B) $24\sqrt{3}$

(C) $12\sqrt{3}$

(D) $\sqrt{3}$

(E) $2\sqrt{3}$