King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics

Math 201

Final Exam, Semester I, 2009-2010

Duration: 180 minutes

Code 001

Name: ________________

Id: ________________

Section: ________________

Do not open this exam until you are told to do so.

Important Instructions:

1. This exam has 11 pages including this cover. There are 20 problems. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
2. All types of calculators, pagers or mobile phones are NOT allowed during the examination.
3. Use HB 2.5 pencils only.
4. Use a good eraser. DO NOT use the eraser attached to the pencil.
5. Write your name, ID number and Section number on the examination paper and in the upper left corner of the answer sheet.
6. When bubbling your ID number and Section number, be sure that the bubbles match with the numbers that you write.
7. The Test Code Number is already bubbled in your answer sheet. Make sure that it is the same as that printed on your question paper.
8. When bubbling, make sure that the bubbled space is fully covered.
9. When erasing a bubble, make sure that you do not leave any trace of penciling

(1) An equation of the plane containing the line L with symmetric equations

$$\frac{x-2}{3} = \frac{1-y}{2} = z+1$$

and the point $P(1,3,1)$ is

(A) $x + 7y - z = 21$

(B) $6x + 7y - 4z = 23$

(C) $6x + y - z = 8$

(D) $2x + 3y - z = 1$

(E) $3x + y + 3z = 4$

(2) The equation of the tangent line to the polar curve

$$r = \cot\theta \quad \text{at} \quad \theta = \frac{\pi}{6} \quad \text{is}$$

(A) $y = \frac{\sqrt{3}}{15}x + \frac{2\sqrt{3}}{5}$

(B) $y = x + \sqrt{3}$

(C) $y = \frac{\sqrt{3}}{5}x + \frac{2\sqrt{3}}{3}$

(D) $y = -2x + 5$

(E) $y = \sqrt{3}$

(3) Let (ρ, θ, ϕ) be the spherical coordinates of the point with rectangular coordinates $(\sqrt{2}, \sqrt{2}, \sqrt{2})$. The value of

$$\rho^2 \sin\theta + 3\cos\phi$$

is equal to

(A) $\frac{\sqrt{2}+\sqrt{3}}{3}$

(B) $2\sqrt{3}$

(C) $\sqrt{3}$

(D) 0

(E) $3\sqrt{2}+\sqrt{3}$

(4) The angle between the planes

$$x - 2y - 2z = 1 \qquad \text{and} \qquad 6x + 3y + 2z = -2,$$

is

(A) $\frac{\pi}{4}$

(B) $\frac{\pi}{6}$

(C) $\cos^{-1}\left(\frac{-4}{21}\right)$

(D) $\cos^{-1}\left(\frac{1}{3}\right)$

(E) $\frac{\pi\sqrt{3}}{3}$

(5) Let

$$f(x, y) = \cos(xy).$$

Suppose x and y are functions of s and t with

$$x(-1,1) = \sqrt{\pi}, \qquad y(-1,1) = \frac{\sqrt{\pi}}{3},$$

$$\frac{\partial x}{\partial s}(-1,1) = 6\sqrt{\pi}, \quad \frac{\partial x}{\partial t}(-1,1) = -2\sqrt{\pi},$$

$$\frac{\partial y}{\partial s}(-1,1) = 2\sqrt{\pi}, \quad \frac{\partial y}{\partial t}(-1,1) = 5\sqrt{\pi}.$$

Then $\frac{\partial f}{\partial s}(-1,1)$ is equal to

(A) π

(B) $-\sqrt{3}\pi$

(C) $\frac{-3\pi}{2}$

(D) $-2\sqrt{3}\pi$

(E) $\frac{\pi}{3}$

(6) Given that the function

$$f(x, y) = (x-1)^2 + (y-1)^2$$

does not have any critical points in the interior of the rectangular domain

$$D = \{(x, y) : 0 \le x \le 1,\ 0 \le y \le 2\},$$

the absolute maximum value of f on D is

(A) 1
(B) 4
(C) $\frac{1}{2}$
(D) 2
(E) 3

(7) The length of the curve given parametrically by

$$x = t^2/2, \qquad y = \frac{1}{3}(2t+1)^{3/2}, \qquad 0 \le t \le 4$$

is

(A) 12

(B) 6

(C) 8

(D) 10

(E) 14

(8) Consider the sphere

$$x^2 + (y-3)^2 + z^2 = 25$$

and the cylinder

$$x^2 + y^2 = 4.$$

The volume of the solid region inside both the sphere and the cylinder is given in cylindrical coordinates by

(A) $\int_0^{2\pi} \int_0^2 \int_{-\sqrt{16r\sin\theta}}^{\sqrt{16-r^2}} r dz dr d\theta$

(B) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_0^1 \int_{-\sqrt{16-r^2+6r\sin\theta}}^{\sqrt{16-r^2+6r\sin\theta}} r dz dr d\theta$

(C) $\int_0^{2\pi} \int_0^2 \int_{-\sqrt{16-r^2+6r\sin\theta}}^{\sqrt{16-r^2+6r\sin\theta}} r dz dr d\theta$

(D) $\int_0^{2\pi} \int_0^2 \int_{-\sqrt{16-r^2+6r\sin\theta}}^{\sqrt{16-r^2+6r\sin\theta}} dr dz d\theta$

(E) $\int_0^1 \int_0^{2\pi} \int_0^{\sqrt{16-r^2+6r\sin\theta}} r dz dr d\theta$

(9) If (a, b) is a critical point of a function f, and if

$$f_{xx}(a,b) = -2 \qquad \text{and} \qquad f_{yy}(a,b) = 3,$$

then what can one say about (a, b)?

(A) f has a local minimum at (a, b).

(B) Nothing can be concluded from the given information.

(C) f has a saddle point at (a, b).

(D) f has a local maximum at (a, b).

(E) $f_{xxyy}(a,b) = -6$

(10) The equation of the tangent plane to the surface given by the equation

$$x\cos(z) - y^2 \sin(xz) = 2$$

at the point $P(2, 1, 0)$ is

(A) $2x + 3y - 2z = 5$

(B) $x - 2z = 2$

(C) $x - 2y = 2$

(D) $x + y - 2z = 3$

(E) $x - y + 2z = 1$

(11) The integral that gives the volume of the solid region inside the sphere centered at the origin with radius 19 and between the cones $\phi = \frac{\pi}{4}$ and $\phi = \frac{\pi}{6}$ is

(A) $\int_0^{2\pi} \int_{\pi/4}^{\pi/6} \int_0^{19/2} \rho \sin\phi \; d\rho d\phi d\theta$

(B) $\int_0^{2\pi} \int_{\pi/6}^{\pi/4} \int_0^{19} \sin^2\phi \; d\rho d\phi d\theta$

(C) $\int_0^{2\pi} \int_{\pi/6}^{\pi/4} \int_0^{19} \rho \cos\phi \; d\rho d\phi d\theta$

(D) $\int_0^{2\pi} \int_{\pi/6}^{\pi/4} \int_0^{19} \rho^2 \sin\phi \; d\rho d\phi d\theta$

(E) $\int_0^{\pi} \int_{\pi/6}^{\pi/4} \int_{-19}^{19} \rho^2 \sin\phi \; d\rho d\phi d\theta$

(12) In an experiment, the temperature of a sample (in degrees Celsius) is given by the function

$$T(x, y, z) = 2y^2 + ze^{-x} + 16,$$

where x, y and z are variables. Using the linear approximation of the function T at the point $(0, 1, 2)$, then $T(0.2, 0.9, 2.3)$ is approximately equal to

(A) 19.5

(B) 19.6

(C) 20.1

(D) 20.2

(E) 19.9

(13) At the point $(0, \pi/4)$, the rate of change of the function

$$f(x, y) = e^{xy} \cos(y)$$

is maximized in the direction of

(A) $\langle 2, 1 \rangle$

(B) $\langle 1, 0 \rangle$

(C) $\langle 4, \pi \rangle$

(D) $\langle 0, 1 \rangle$

(E) $\langle \pi, -4 \rangle$

(14) The maximum of

$$f(x, y) = xy$$

subject to the constraint

$$(x + 1)^2 + y^2 = 1$$

is equal to

(A) 0

(B) $\sqrt{3}$

(C) $\frac{3\sqrt{3}}{4}$

(D) $\frac{3}{4}$

(E) $\frac{3\sqrt{3}}{8}$

(15) Let E be the solid region that lies under the plane $z = 1 + x + y$ and above the region of the xy–plane in the first quadrant bounded by the curves $x = y^2$, $y = 0$, and $x = 1$. Then

$$\iiint_E 2xy dV$$

is equal to

(A) $\frac{65}{28}$

(B) $\frac{65}{84}$

(C) $\frac{65}{42}$

(D) $\frac{65}{12}$

(E) $\frac{65}{14}$

(16) The value of the double integral

$$\int_0^1 \int_{\sqrt{x}}^1 \cos(y^3) dy dx$$

is equal to

(A) $\frac{\sin 1}{3}$

(B) $\frac{\sin 1}{4}$

(C) $\frac{\cos 1}{3}$

(D) $\frac{4 \cos 1}{3}$

(E) $\frac{\sin 1 \cos 1}{3}$

(17) Let D be the disc centered at the point $(1, 0)$ with radius 1. Using polar coordinates, the value of the double integral

$$\iint_D \sqrt{x^2+y^2}dA$$

is equal to

(A) $\frac{4}{3}\int_{-\pi/2}^{\pi/2} \cos^2\theta \, d\theta$

(B) $\frac{3}{8}\int_{0}^{\pi/2} \cos^3\theta \, d\theta$

(C) 0

(D) $\frac{16}{3}\int_{0}^{\pi/2} \cos^3\theta \, d\theta$

(E) $\frac{16}{3}\int_{0}^{1} r^3 dr$

(18) The volume of the solid that lies under the graph of

$$f(x, y) = xe^y$$

and above the triangular region with vertices $(0, 0)$, $(2, 2)$, $(4, 0)$ is equal to

(A) $3\left(e^2 - 4\right)$

(B) $4e - 3$

(C) e^2

(D) $8e$

(E) $4(e^2 - 3)$

(19) The equation

$$z^2 = r\left(r\sin^2\theta - \cos\theta\right)$$

represents

(A) A hyperboloid of two sheets

(B) A hyperboloid of one sheet

(C) A hyperbolic paraboloid

(D) An elliptic paraboloid

(E) A cone

(20) The area of the parallelogram with adjacent sides

$$\overrightarrow{u} = \langle 1, 1, -1 \rangle \quad \text{and} \quad \overrightarrow{v} = \langle 2, 1, 1 \rangle$$

is equal to

(A) $\sqrt{14}$

(B) $\sqrt{10}$

(C) 3

(D) $\sqrt{22}$

(E) 2

Answer Key Final Math 201-091
Version1

1.	B
2.	A
3.	E
4.	C
5.	D
6.	D
7.	A
8.	C
9.	C
10.	B
11.	D
12.	A
13.	E
14.	C
15.	B
16.	A
17.	D
18.	E
19.	C
20.	A