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Nondifferentiable second order symmetric duality in multiobjective programming

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Abstract

A pair of Mond–Weir type nondifferentiable multiobjective second order symmetric dual programs is formulated and symmetric duality theorems are established under the assumptions of second order F-pseudoconvexity/F-pseudoconcavity.

Keywords: Symmetric duality; Multiobjective programming; Second order F-pseudoconvexity; Efficient solutions

1. Introduction

Symmetric duality in mathematical programming was introduced by Dorn [8], who defined a program and its dual to be symmetric if the dual of the dual is the original problem. Chandra and Husain [5] studied a pair of symmetric dual nondifferentiable programs by assuming convexity/concavity of the scalar function $f(x, y)$. Subsequently, Chandra et al. [3] presented another pair of symmetric dual nondifferentiable programs weakening convexity/concavity assumptions to pseudoconvexity/pseudoconcavity.

Weir and Mond [18] discussed symmetric duality in multiobjective programming by using the concept of efficiency. Chandra and Prasad [6] presented a pair of multiobjective programming problems by associating a vector valued infinite game to this pair. Gulati et al. [10] also established duality results for multiobjective symmetric dual problems without non-negativity constraints.

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Mangasarian [12] considered a nonlinear program and discussed second order duality under certain inequalities. Mond [14] assumed rather simple inequalities. Bector and Chandra [2] defined the functions satisfying the inequalities in [14] to be bonvex/boncave. Mangasarian [12, p. 609] and Mond [14, p. 93] have indicated possible computational advantages of second order duals over the first order duals. An alternative approach to higher order duality is given in [17].

Mishra [13] and Gulati et al. [9] studied single objective second order symmetric duality for Wolfe and Mond–Weir type models. Kim et al. [11] presented a pair of multiobjective second order symmetric dual problems and established duality results under convexity. Recently, Yang and Hou [19] applied invexity to multiobjective second order symmetric dual problems of [11] omitting non-negativity constraints but with an additional assumption on the invexity.

In this paper, we consider a pair of nondifferentiable multiobjective second order symmetric dual programs of Mond–Weir type. We prove weak, strong and converse duality theorems under second order F-pseudoconvexity/F-pseudoconcavity.

2. Notations and preliminaries

The following convention for inequalities will be used: If $x, u \in R^n$, then $x \geqq u \Leftrightarrow x_i \geqq u_i$, $i = 1, 2, \ldots, n$; $x \geq u \Leftrightarrow x \geqq u$ and $x \neq u$; $x > u \Leftrightarrow x_i > u_i$, $i = 1, 2, \ldots, n$.

If $g(x, y)$ is a real valued twice differentiable function of x and y, where $x \in R^n$ and $y \in R^m$, then $\nabla_x g(\bar{x}, \bar{y})$ and $\nabla_y g(\bar{x}, \bar{y})$ denote the gradient vectors with respect to first and second variable evaluated at $(\bar{x}, \bar{y})$ respectively. Also $\nabla_{xx} g(\bar{x}, \bar{y})$ and $\nabla_{yy} g(\bar{x}, \bar{y})$ are, respectively, the $n \times n$ and $m \times m$ symmetric Hessian matrices with respect to first and second variable evaluated at $(\bar{x}, \bar{y})$.

Consider the following multiobjective programming problem:

(P) Minimize $f(x)$

subject to $x \in X$,

where $f : R^n \to R^k$ and $X \subset R^n$.

Definition 2.1. A point $\bar{x} \in X$ is said to be weak efficient for (P) if there exists no other $x \in X$ with $f(x) < f(\bar{x})$.

Definition 2.2. A point $\bar{x} \in X$ is said to be an efficient solution of (P) if there exists no other $x \in X$ such that $f(x) \leq f(\bar{x})$.

Definition 2.3. A functional $F : X \times X \times R^n \longrightarrow R$ (where $X \subseteq R^n$) is sublinear in its third component, if for all $x, u \in X$,

(i) $F(x, u; a_1 + a_2) \leqq F(x, u; a_1) + F(x, u; a_2)$ for all $a_1, a_2 \in R^n$; and
(ii) $F(x, u; \alpha a) = \alpha F(x, u; a)$ for all $\alpha \in R_+$, and for all $a \in R^n$.

For notational convenience, we write

$$F_{x,u}(a) = F(x, u; a).$$

Definition 2.4. A real valued twice differentiable function $g(., y) : X \times Y \to R$ is said to be second order F-pseudoconvex at $u \in X$ with respect to $p \in R^n$, if there exists a sublinear functional $F : X \times X \times R^n \to R$ such that

$$F_{x,u}(\nabla_x g(u, y) + \nabla_{xx} g(u, y)p) \geqq 0 \Rightarrow g(x, y) \geqq g(u, y) - \frac{1}{2} p^t \nabla_{xx} g(u, y)p.$$

A real valued twice differentiable function g is second order F-pseudoconcave if $-g$ is second order F-pseudoconvex.

We shall make use of the following generalized Schwartz inequality:

$$x^t Ay \leqq (x^t Ax)^{\frac{1}{2}} (y^t Ay)^{\frac{1}{2}}$$

where $x, y \in R^n$, and $A \in R^n \times R^n$ is a positive semidefinite matrix. Equality holds if for some $\lambda \geqq 0$, $Ax = \lambda Ay$.

3. Mond–Weir type second order symmetric duality

We present the following pair of second order nondifferentiable multiobjective problems with k-objectives and establish weak, strong and converse duality theorems.

(MP): Minimize $K(x, y, w, p) = (K_1(x, y, w, p), K_2(x, y, w, p), \ldots, K_k(x, y, w, p))$

$$\text{subject to} \quad \sum_{i=1}^{k} \lambda_i [\nabla_y f_i(x, y) - C_i w_i + \nabla_{yy} f_i(x, y) p_i] \leqq 0 \tag{1}$$

$$y^t \sum_{i=1}^{k} \lambda_i [\nabla_y f_i(x, y) - C_i w_i + \nabla_{yy} f_i(x, y) p_i] \geqq 0 \tag{2}$$

$$w_i^t C_i w_i \leqq 1, i = 1, 2, \ldots, k \tag{3}$$

$$\lambda > 0 \tag{4}$$

$$x \geqq 0. \tag{5}$$

(MD): Maximize $G(u, v, z, r) = (G_1(u, v, z, r), G_2(u, v, z, r), \ldots, G_k(u, v, z, r))$

$$\text{subject to} \quad \sum_{i=1}^{k} \lambda_i [\nabla_x f_i(u, v) + B_i z_i + \nabla_{xx} f_i(u, v) r_i] \geqq 0 \tag{6}$$

$$u^t \sum_{i=1}^{k} \lambda_i [\nabla_x f_i(u, v) + B_i z_i + \nabla_{xx} f_i(u, v) r_i] \leqq 0 \tag{7}$$

$$z_i^t B_i z_i \leqq 1, i = 1, 2, \ldots, k \tag{8}$$

$$\lambda > 0 \tag{9}$$

$$v \geqq 0. \tag{10}$$

where

$$K_i(x, y, w, p) = f_i(x, y) + (x^t B_i x)^{\frac{1}{2}} - y^t C_i w_i - \frac{1}{2} p_i^t \nabla_{yy} f_i(x, y) p_i,$$

$$G_i(u, v, z, r) = f_i(u, v) - (v^t C_i v)^{\frac{1}{2}} + u^t B_i z_i - \frac{1}{2} r_i^t \nabla_{xx} f_i(u, v) r_i,$$

$\lambda_i \in R$, $p_i \in R^m$, $r_i \in R^n$, $i = 1, 2, \ldots, k$, and $f_i, i = 1, 2, \ldots, k$ are thrice differentiable functions from $R^n \times R^m$ to R, B_i and $C_i, i = 1, 2, \ldots, k$ are positive semidefinite matrices. Also we take $p = (p_1, p_2, \ldots, p_k)$, $r = (r_1, r_2, \ldots, r_k)$, $w = (w_1, w_2, \ldots, w_k)$ and $z = (z_1, z_2, \ldots, z_k)$.

Remark. If $k = 1$, then (MP) and (MD) become the nondifferentiable second order symmetric dual programs of Ahmad and Husain [1].

Theorem 3.1 (*Weak Duality*). *Let (x, y, λ, w, p) be feasible for (MP) and (u, v, λ, z, r) feasible for (MD). Assume that*

(i) $\sum_{i=1}^{k} \lambda_i [f_i(., v) + (.)^t B_i z_i]$ *is second order F-pseudoconvex at u,*
(ii) $\sum_{i=1}^{k} \lambda_i [f_i(x, .) - (.)^t C_i w_i]$ *is second order F-pseudoconcave at y,*
(iii) $F_{x,u}(\xi) + u^t \xi \geqq 0$, *for $\xi \in R^n$, and*
(iv) $F_{v,y}(\eta) + y^t \eta \geqq 0$, *for $\eta \in R^m$. Then*

$$K(x, y, w, p) \nleq G(u, v, z, r).$$

Proof. By taking $\xi = \sum_{i=1}^{k} \lambda_i (\nabla_x f_i(u, v) + B_i z_i + \nabla_{xx} f_i(u, v) r_i)$, we have

$$F_{x,u}\left(\sum_{i=1}^{k} \lambda_i (\nabla_x f_i(u, v) + B_i z_i + \nabla_{xx} f_i(u, v) r_i)\right)$$
$$\geqq -u^t \left(\sum_{i=1}^{k} \lambda_i (\nabla_x f_i(u, v) + B_i z_i + \nabla_{xx} f_i(u, v) r_i)\right) \geqq 0 \text{ (by hypothesis (iii) and (7))},$$

which by second order F-pseudoconvexity of $\sum_{i=1}^{k} \lambda_i [f_i(., v) + (.)^t B_i z_i]$ at u yields

$$\sum_{i=1}^{k} \lambda_i [f_i(x, v) + x^t B_i z_i] \geqq \sum_{i=1}^{k} \lambda_i \left[f_i(u, v) + u^t B_i z_i - \frac{1}{2} r_i^t \nabla_{xx} f_i(u, v) r_i \right]. \tag{11}$$

On taking $\eta = -\sum_{i=1}^{k} \lambda_i (\nabla_y f_i(x, y) - C_i w_i + \nabla_{yy} f_i(x, y) p_i)$, we have

$$F_{v,y}\left(-\sum_{i=1}^{k} \lambda_i (\nabla_y f_i(x, y) - C_i w_i + \nabla_{yy} f_i(x, y) p_i)\right)$$
$$\geqq y^t \left(\sum_{i=1}^{k} \lambda_i \left(\nabla_y f_i(x, y) - C_i w_i + \nabla_{yy} f_i(x, y) p_i\right)\right) \geqq 0 \text{ (by hypothesis (iv) and (2))},$$

which by second order F-pseudoconcavity of $\sum_{i=1}^{k} \lambda_i [f_i(x, .) - (.)^t C_i w_i]$ at y gives

$$\sum_{i=1}^{k} \lambda_i [f_i(x, v) - v^t C_i w_i] \leqq \sum_{i=1}^{k} \lambda_i \left[f_i(x, y) - y^t C_i w_i - \frac{1}{2} p_i^t \nabla_{yy} f_i(x, y) p_i \right]. \tag{12}$$

Combining inequalities (11) and (12), we get

$$\sum_{i=1}^{k} \lambda_i [x^t B_i z_i + v^t C_i w_i] \geqq \sum_{i=1}^{k} \lambda_i \left[f_i(u, v) + u^t B_i z_i - \frac{1}{2} r_i^t \nabla_{xx} f_i(u, v) r_i \right.$$
$$\left. - f_i(x, y) + y^t C_i w_i + \frac{1}{2} p_i^t \nabla_{yy} f_i(x, y) p_i \right].$$

Applying the Schwartz inequality, (3) and (8), we obtain

$$\sum_{i=1}^{k} \lambda_i \left[f_i(x, y) + (x^t B_i x)^{\frac{1}{2}} - y^t C_i w_i - \frac{1}{2} p_i^t \nabla_{yy} f_i(x, y) p_i \right]$$
$$\geqq \sum_{i=1}^{k} \lambda_i \left[f_i(u, v) - (v^t C_i v)^{\frac{1}{2}} + u^t B_i z_i - \frac{1}{2} r_i^t \nabla_{xx} f_i(u, v) r_i \right].$$

Hence

$$K(x, y, w, p) \nleq G(u, v, z, r). \quad \square$$

Theorem 3.2 (*Strong Duality*). *Let f be thrice differentiable on $R^n \times R^m$ and $(\bar{x}, \bar{y}, \bar{\lambda}, \bar{w}, \bar{p})$, a weak efficient solution for (MP), and $\lambda = \bar{\lambda}$ fixed in (MD). Assume that*

(i) $\nabla_{yy} f_i$ *is nonsingular for all* $i = 1, 2, \ldots, k$,

(ii) *the matrix* $\sum_{i=1}^{k} \bar{\lambda}_i (\nabla_{yy} f_i \bar{p}_i)_y$ *is positive or negative definite, and*

(iii) *the set* $\{\nabla_y f_1 - C_1 \bar{w}_1 + \nabla_{yy} f_1 \bar{p}_1, \nabla_y f_2 - C_2 \bar{w}_2 + \nabla_{yy} f_2 \bar{p}_2, \ldots, \nabla_y f_k - C_k \bar{w}_k + \nabla_{yy} f_k \bar{p}_k\}$ *is linearly independent,*

where $f_i = f_i(\bar{x}, \bar{y})$, $i = 1, 2, \ldots, k$. Then $(\bar{x}, \bar{y}, \bar{\lambda}, \bar{z}, \bar{r} = 0)$ is feasible for (MD), and the two objectives have the same values. Also, if the hypotheses of Theorem 3.1 are satisfied for all feasible solutions of (MP) and (MD), then $(\bar{x}, \bar{y}, \bar{\lambda}, \bar{z}, \bar{r} = 0)$ is an efficient solution for (MD).

Proof. Since $(\bar{x}, \bar{y}, \bar{\lambda}, \bar{w}, \bar{p})$ is a weak efficient solution of (MP), by the Fritz–John conditions [7], there exist $\alpha \in R^k$, $\beta \in R^m$, $\gamma \in R$, $\nu \in R^k$, $\delta \in R^k$ and $\xi \in R^n$ such that

$$\sum_{i=1}^{k} \alpha_i \left[\nabla_x f_i + B_i \bar{z}_i - \frac{1}{2} (\nabla_{yy} f_i \bar{p}_i)_x \bar{p}_i \right]$$
$$+ \sum_{i=1}^{k} \bar{\lambda}_i \left[\nabla_{yx} f_i + (\nabla_{yy} f_i \bar{p}_i)_x \right] (\beta - \gamma \bar{y}) - \xi = 0 \tag{13}$$

$$\sum_{i=1}^{k} \alpha_i \left[\nabla_y f_i - C_i \bar{w}_i - \frac{1}{2} (\nabla_{yy} f_i \bar{p}_i)_y \bar{p}_i \right]$$
$$+ \sum_{i=1}^{k} \bar{\lambda}_i [\nabla_{yy} f_i + (\nabla_{yy} f_i \bar{p}_i)_y](\beta - \gamma \bar{y}) - \gamma \sum_{i=1}^{k} \bar{\lambda}_i [\nabla_y f_i - C_i \bar{w}_i + \nabla_{yy} f_i \bar{p}_i] = 0 \tag{14}$$

$$(\beta - \gamma \bar{y})^t \left[\nabla_y f_i - C_i \bar{w}_i + \nabla_{yy} f_i \bar{p}_i \right] - \delta_i = 0, \qquad i = 1, 2, \ldots, k \tag{15}$$

$$\alpha_i C_i \bar{y} + (\beta - \gamma \bar{y})^t \bar{\lambda}_i C_i = 2 \nu_i C_i \bar{w}_i, \qquad i = 1, 2, \ldots, k \tag{16}$$

$$\left[(\beta - \gamma \bar{y}) \bar{\lambda}_i - \alpha_i \bar{p}_i \right]^t \nabla_{yy} f_i = 0, \qquad i = 1, 2, \ldots, k \tag{17}$$

$$\bar{x}^t B_i \bar{z}_i = (\bar{x}^t B_i \bar{x})^{\frac{1}{2}}, \qquad i = 1, 2, \ldots, k \tag{18}$$

$$\beta^t \sum_{i=1}^{k} \bar{\lambda}_i (\nabla_y f_i - C_i \bar{w}_i + \nabla_{yy} f_i \bar{p}_i) = 0 \tag{19}$$

$$\gamma \bar{y} \sum_{i=1}^{k} \bar{\lambda}_i (\nabla_y f_i - C_i \bar{w}_i + \nabla_{yy} f_i \bar{p}_i) = 0 \tag{20}$$

$$\nu_i (\bar{w}_i^t C_i \bar{w}_i - 1) = 0, \qquad i = 1, 2, \dots, k \tag{21}$$

$$\delta^t \bar{\lambda} = 0 \tag{22}$$

$$\bar{x}^t \xi = 0 \tag{23}$$

$$\bar{z}_i^t B_i \bar{z}_i \leqq 1, \qquad i = 1, 2, \dots, k \tag{24}$$

$$(\alpha, \beta, \gamma, \nu, \delta, \xi) \geqq 0 \tag{25}$$

$$(\alpha, \beta, \gamma, \nu, \delta, \xi) \neq 0. \tag{26}$$

Since $\bar{\lambda} > 0$ and $\delta \geqq 0$, (22) implies $\delta = 0$. Consequently, (15) yields

$$(\beta - \gamma \bar{y})^t \left[\nabla_y f_i - C_i \bar{w}_i + \nabla_{yy} f_i \bar{p}_i\right] = 0, \qquad i = 1, 2, \dots, k. \tag{27}$$

Since $\nabla_{yy} f_i$ is nonsingular for $i = 1, 2, \dots, k$, from (17), it follows that

$$(\beta - \gamma \bar{y}) \bar{\lambda}_i = \alpha_i \bar{p}_i, \qquad i = 1, 2, \dots, k. \tag{28}$$

From (14), we get

$$\sum_{i=1}^{k} (\alpha_i - \gamma \bar{\lambda}_i)(\nabla_y f_i - C_i \bar{w}_i) + \sum_{i=1}^{k} \bar{\lambda}_i \nabla_{yy} f_i (\beta - \gamma \bar{y} - \gamma \bar{p}_i)$$
$$+ \sum_{i=1}^{k} (\nabla_{yy} f_i \bar{p}_i)_y \left[(\beta - \gamma \bar{y}) \bar{\lambda}_i - \frac{1}{2} \alpha_i \bar{p}_i\right] = 0.$$

By using (28), it follows that

$$\sum_{i=1}^{k} (\alpha_i - \gamma \bar{\lambda}_i)(\nabla_y f_i - C_i \bar{w}_i + \nabla_{yy} f_i \bar{p}_i) + \frac{1}{2} \sum_{i=1}^{k} \bar{\lambda}_i (\nabla_{yy} f_i \bar{p}_i)_y (\beta - \gamma \bar{y}) = 0. \tag{29}$$

Premultiplying (29) by $(\beta - \gamma \bar{y})^t$ and using (27), we obtain

$$(\beta - \gamma \bar{y})^t \sum_{i=1}^{k} \bar{\lambda}_i (\nabla_{yy} f_i \bar{p}_i)_y (\beta - \gamma \bar{y}) = 0,$$

which by hypothesis (ii) implies

$$\beta = \gamma \bar{y}. \tag{30}$$

Therefore, from (29), we get $\sum_{i=1}^{k} (\alpha_i - \gamma \bar{\lambda}_i)(\nabla_y f_i - C_i \bar{w}_i + \nabla_{yy} f_i \bar{p}_i) = 0$, which by hypothesis (iii) yields

$$\alpha_i = \gamma \bar{\lambda}_i, \qquad i = 1, 2, \dots, k. \tag{31}$$

If $\gamma = 0$, then $\alpha_i = 0, i = 1, 2, \dots, k$ and from (30), $\beta = 0$. Also from (13) and (16), we get $\xi_i = 0, \nu_i = 0, i = 1, 2, \dots, k$. Thus $(\alpha, \beta, \gamma, \delta, \nu, \xi) = 0$, a contradiction to (26). Hence $\gamma > 0$. Since $\bar{\lambda}_i > 0, i = 1, 2, \dots, k$, (31) implies $\alpha_i > 0, i = 1, 2, \dots, k$. Using (30) in (28), we get $\alpha_i \bar{p}_i = 0, i = 1, 2, \dots, k$, and hence $\bar{p}_i = 0, i = 1, 2, \dots, k$. Using (30) and $\bar{p}_i = 0, i = 1, 2, \dots, k$, in (13), it follows that $\sum_{i=1}^{k} \alpha_i [\nabla_x f_i + B_i \bar{z}_i] = \xi$, which by (31) gives

$$\sum_{i=1}^{k} \bar{\lambda}_i [\nabla_x f_i + B_i \bar{z}_i] = \frac{\xi}{\gamma} \geqq 0, \tag{32}$$

and

$$\bar{x}^t \sum_{i=1}^{k} \bar{\lambda}_i [\nabla_x f_i + B_i \bar{z}_i] = \frac{\bar{x}^t \xi}{\gamma} = 0. \tag{33}$$

Also, from (30), we have

$$\bar{y} = \frac{\beta}{\gamma} \geqq 0. \tag{34}$$

Hence from (24) and (32)–(34), $(\bar{x}, \bar{y}, \bar{\lambda}, \bar{z}, \bar{r} = 0)$ is feasible for (MD). Now let $\frac{2\nu_i}{\alpha_i} = a$. Then $a \geqq 0$ and from (16) and (30)

$$C_i \bar{y} = a C_i \bar{w}_i, \tag{35}$$

which is the condition for equality in the Schwartz inequality. Therefore

$$\bar{y}^t C_i \bar{w}_i = (\bar{y}^t C_i \bar{y})^{\frac{1}{2}} (\bar{w}_i^t C_i \bar{w}_i)^{\frac{1}{2}}.$$

In case $\nu_i > 0$, (21) gives $\bar{w}_i^t C_i \bar{w}_i = 1$ and so $\bar{y}^t C_i \bar{w}_i = (\bar{y}^t C_i \bar{y})^{\frac{1}{2}}$. In case $\nu_i = 0$, (35) gives $C_i \bar{y} = 0$ and so $\bar{y}^t C_i \bar{w}_i = (\bar{y}^t C_i \bar{y})^{\frac{1}{2}} = 0$. Thus in either case

$$\bar{y}^t C_i \bar{w}_i = (\bar{y}^t C_i \bar{y})^{\frac{1}{2}}. \tag{36}$$

Hence

$$\begin{aligned} K_i(\bar{x}, \bar{y}, \bar{w}, \bar{p} = 0) &= f_i(\bar{x}, \bar{y}) + (\bar{x}^t B_i \bar{x})^{\frac{1}{2}} - \bar{y}^t C_i \bar{w}_i \\ &= f_i(\bar{x}, \bar{y}) - (\bar{y}^t C_i \bar{y})^{\frac{1}{2}} + \bar{x}^t B \bar{z}_i = G_i(\bar{x}, \bar{y}, \bar{z}, \bar{r} = 0) \text{ (using (18) and (36))}. \end{aligned}$$

Now it follows from Theorem 3.1 that $(\bar{x}, \bar{y}, \bar{\lambda}, \bar{z}, \bar{r} = 0)$ is an efficient solution for (MD).

A converse duality theorem may be merely stated as its proof would run analogously to that of Theorem 3.2. □

Theorem 3.3 (*Converse Duality*). *Let f be thrice differentiable on $R^n \times R^m$ and $(\bar{u}, \bar{v}, \bar{\lambda}, \bar{z}, \bar{r})$, a weak efficient solution for (MD), and $\lambda = \bar{\lambda}$ fixed in (MP). Assume that*

(i) *$\nabla_{xx} f_i$ is nonsingular for all $i = 1, 2, \ldots, k$,*
(ii) *the matrix $\sum_{i=1}^{k} \bar{\lambda}_i (\nabla_{xx} f_i \bar{r}_i)_x$ is positive or negative definite, and*
(iii) *the set $\{\nabla_x f_1 + B_1 \bar{z}_1 + \nabla_{xx} f_1 \bar{r}_1, \nabla_x f_2 + B_2 \bar{z}_2 + \nabla_{xx} f_2 \bar{r}_2, \ldots, \nabla_x f_k + B_k \bar{z}_k + \nabla_{xx} f_k \bar{r}_k\}$ is linearly independent,*

where $f_i = f_i(\bar{u}, \bar{v}), i = 1, 2, \ldots, k$. Then $(\bar{u}, \bar{v}, \bar{\lambda}, \bar{w}, \bar{p} = 0)$ is feasible for (MP), and the two objectives have the same values. Also, if the hypotheses of Theorem 3.1 are satisfied for all feasible solutions of (MP) and (MD), then $(\bar{u}, \bar{v}, \bar{\lambda}, \bar{w}, \bar{p} = 0)$ is an efficient solution for (MP).

4. Special cases

(i) If $B_i = C_i = 0, i = 1, 2, \ldots, k$, then (MP) and (MD) reduce to the second order multiobjective symmetric dual program studied by Suneja et al. [16] with the omission of non-negativity constraints from (MP) and (MD). If in addition $p = r = 0$, and $k = 1$, then we get the first order symmetric dual programs of Chandra et al. [4].

(ii) If we set $p = r = 0$, in (MP) and (MD), then we obtain a pair of first order symmetric dual nondifferentiable multiobjective programs considered by Mond et al. [15].

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