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Second order (F, α, ρ, d)-convexity and duality in multiobjective programming

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Abstract

A class of second order (F, α, ρ, d)-convex functions and their generalizations is introduced. Using the assumptions on the functions involved, weak, strong and strict converse duality theorems are established for a second order Mond–Weir type multiobtive dual.

Keywords: Multiobjective programming; Second order duality; Efficient solutions; Generalized (F, α, ρ, d)-convexity

1. Introduction

The importance of convex functions is well known in optimization theory. But for many mathematical models used in decision sciences, economics, management sciences, stochastics, applied mathematics and engineering, the notion

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of convexity does no longer suffice. So it is possible to generalize the notion of convexity and to extend the validity of results to larger classes of optimization problems. Consequently, various generalizations of convex functions have been introduced in the literature. More specifically, the concept of (F, ρ)-convexity was introduced by Preda [13], an extension of F-convexity defined by Hanson and Mond [6] and ρ-convexity given by Vial [14]. Gulati and Islam [5] and Ahmad [2] established optimality conditions and duality results for multiobjective programming problems involving F-convexity and (F, ρ)-convexity assumptions, respectively.

Duality theory has been of much interest and many contributions [1,3,4,7,9–11,16] have been made to its development. Mangasarian [10] first formulated the second order dual for a nonlinear programming problem and established duality results under somewhat involved assumptions. Mond [11] reproved second order duality theorems under simpler assumptions than those previously given by Mangasarian [10], and showed that the second order dual has computational advantages over the first order dual. Recently, Yang et al. [15] formulated several second order duals for scalar programming problem and discussed duality results involving generalized F-convex functions.

Zhang and Mond [16] extended the class of (F, ρ)-convex functions to second order (F, ρ)-convex functions and obtained duality results for Mangasarian type, Mond–Weir type and general Mond–Weir type multiobjective dual problems. Recently, Aghezzaf [1] formulated a mixed type dual for multiobjective programming problem and discussed various duality results by defining new classes of generalized second order (F, ρ)-convexity for multiobjective functions.

Motivated by various concepts of generalized convexity, Liang et al. [8,9] introduced a unified formulation of generalized convexity, called (F, α, ρ, d)-convexity and obtained some optimality conditions and duality results for the single objective fractional problems and multiobjective problems.

In this paper, motivated by Liang et al. [8] and Aghezzaf [1], we introduce second order (F, α, ρ, d)-convex functions and their generalizations. These concepts are then used to develop weak, strong and strict converse duality theorems for second order Mond–Weir type multiobjective dual.

2. Notations and preliminaries

Throughout the paper, following convention for vectors in R^n will be followed: $x \geqq y$ if and only if $x_i \geqq y_i$, $i = 1, 2, \ldots, n$, $x \geq y$ if and only if $x \geqq y$ and $x \neq y$, $x > y$ if and only if $x_i > y_i$, $i = 1, 2, \ldots, n$.

The problem to be considered here is the following multiobjective nonlinear programming problem:

$$(\mathbf{P}) \quad \text{Minimize} \quad f(x)$$
$$\text{subject to} \quad g(x) \leqq 0, \quad x \in X, \tag{1}$$

where $f = (f_1, f_2, \ldots, f_k): X \mapsto R^k$, $g = (g_1, g_2, \ldots, g_m): X \mapsto R^m$ are assumed to be twice differentiable functions over X, an open subset of R^n.

Definition 1. A feasible point $\bar{x}$ is said to be an efficient solution of the vector minimum problem (P) if there exists no other feasible point x such that

$$f(x) \leq f(\bar{x}).$$

In the sequel, we require the following definition of sublinear functional:

Definition 2. A functional $F: X \times X \times R^n \mapsto R$ is said to be sublinear in its third component, if for all $x, \bar{x} \in X$

(i) $F(x, \bar{x}; a + b) \leqq F(x, \bar{x}; a) + F(x, \bar{x}; b)$, $\forall a, b \in R^n$,
(ii) $F(x, \bar{x}; \beta a) = \beta F(x, \bar{x}; a)$, $\forall \beta \in R$, $\beta \geqq 0$, and $\forall a \in R^n$.

Let F be sublinear and the function $f = (f_1, f_2, \ldots, f_k): X \mapsto R^k$ be differentiable at $\bar{x} \in X$ and $\rho = (\rho_1, \rho_2, \ldots, \rho_k) \in R^k$.

Definition 3. A twice differentiable function f_i over X is said to be second order (F, α, ρ_i, d)-convex at $\bar{x}$ on X, if for all $x \in X$, there exist vector $p \in R^n$, a real valued function $\alpha: X \times X \mapsto R_+ \backslash \{0\}$, a real valued function $d(\cdot,\cdot): X \times X \mapsto R$ and a real number ρ_i such that

$$f_i(x) - f_i(\bar{x}) + \frac{1}{2} p^t \nabla^2 f_i(\bar{x}) p \geqq F\left(x, \bar{x}; \alpha(x, \bar{x}) \left\{ \nabla f_i(\bar{x}) + \nabla^2 f_i(\bar{x}) p \right\}\right) + \rho_i d^2(x, \bar{x}).$$

A twice differentiable vector function $f: X \mapsto R^k$ is said to be second order (F, α, ρ, d)-convex at $\bar{x}$, if each of its components f_i is second order (F, α, ρ_i, d)-convex at $\bar{x}$.

Definition 4. A twice differentiable function f_i over X is said to be second order (F, α, ρ_i, d)-pseudoconvex at $\bar{x}$ on X, if for all $x \in X$, there exist vector $p \in R^n$, a real valued function $\alpha: X \times X \mapsto R_+ \backslash \{0\}$, a real valued function $d(\cdot,\cdot): X \times X \mapsto R$ and a real number ρ_i such that

$$f_i(x) < f_i(\bar{x}) - \frac{1}{2} p^t \nabla^2 f_i(\bar{x}) p$$
$$\Rightarrow \quad F\left(x, \bar{x}; \alpha(x, \bar{x}) \left\{ \nabla f_i(\bar{x}) + \nabla^2 f_i(\bar{x}) p \right\}\right) < -\rho_i d^2(x, \bar{x}).$$

A twice differentiable vector function $f: X \mapsto R^k$ is said to be second order (F, α, ρ, d)-pseudoconvex at $\bar{x}$, if each of its components f_i is second order (F, α, ρ_i, d)-pseudoconvex at $\bar{x}$.

Definition 5. A twice differentiable function f_i over X is said to be strictly second order (F,α,ρ_i,d)-pseudoconvex at $\bar{x}$ on X, if for all $x\in X$, there exist vector $p\in R^n$, a real valued function $\alpha{:}X\times X\mapsto R_+\backslash\{0\}$, a real valued function $d(\cdot,\cdot){:}X\times X\mapsto R$ and a real number ρ_i such that

$$F(x,\bar{x};\alpha(x,\bar{x})\{\nabla f_i(\bar{x})+\nabla^2 f_i(\bar{x})p\})\geqq-\rho_i d^2(x,\bar{x})$$
$$\Rightarrow\quad f_i(x)>f_i(\bar{x})-\frac{1}{2}p^{\mathrm{t}}\nabla^2 f_i(\bar{x})p,$$

or equivalently

$$f_i(x)\leqq f_i(\bar{x})-\frac{1}{2}p^{\mathrm{t}}\nabla^2 f_i(\bar{x})p$$
$$\Rightarrow\quad F\big(x,\bar{x};\alpha(x,\bar{x})\big\{\nabla f_i(\bar{x})+\nabla^2 f_i(\bar{x})p\big\}\big)<-\rho_i d^2(x,\bar{x}).$$

A twice differentiable vector function $f{:}\,X\mapsto R^k$ is said to be strictly second order (F,α,ρ,d)-pseudoconvex at $\bar{x}$, if each of its components f_i is strictly second order (F,α,ρ_i,d)-pseudoconvex at $\bar{x}$.

Definition 6. A twice differentiable function f_i over X is said to be second order (F,α,ρ_i,d)-quasiconvex at $\bar{x}$ on X, if for all $x\in X$, there exist vector $p\in R^n$, a real valued function $\alpha{:}\,X\times X\mapsto R_+\backslash\{0\}$, a real valued function $d(\cdot,\cdot){:}\,X\times X\mapsto R$ and a real number ρ_i such that

$$f_i(x)\leqq f_i(\bar{x})-\frac{1}{2}p^{\mathrm{t}}\nabla^2 f_i(\bar{x})p$$
$$\Rightarrow\quad F\big(x,\bar{x};\alpha(x,\bar{x})\big\{\nabla f_i(\bar{x})+\nabla^2 f_i(\bar{x})p\big\}\big)\leqq-\rho_i d^2(x,\bar{x}).$$

A twice differentiable vector function $f{:}\,X\mapsto R^k$ is said to be second order (F,α,ρ,d)-quasiconvex at $\bar{x}$, if each of its components f_i is second order (F,α,ρ_i,d)-quasiconvex at $\bar{x}$.

Definition 7. A twice differentiable vector function f over X is said to be strong second order (F,α,ρ,d)-pseudoconvex at $\bar{x}$ on X, if for all $x\in X$, there exist vector $p\in R^n$, a real valued function $\alpha{:}\,X\times X\mapsto R_+\backslash\{0\}$, a real valued function $d(\cdot,\cdot){:}X\times X\mapsto R$ and a vector $\rho\in R^k$ such that

$$f(x)\leq f(\bar{x})-\frac{1}{2}p^{\mathrm{t}}\nabla^2 f(\bar{x})p$$
$$\Rightarrow\quad F\big(x,\bar{x};\alpha(x,\bar{x})\big\{\nabla f(\bar{x})+\nabla^2 f(\bar{x})p\big\}\big)\leq-\rho d^2(x,\bar{x}).$$

The following convention will be followed. If f is an k-dimensional vector function, then $F\big(x,\bar{x};\nabla f(\bar{x})+\nabla^2 f(\bar{x})p\big)$ denotes the vector of components $F\big(x,\bar{x};\nabla f_1(\bar{x})+\nabla^2 f_1(\bar{x})p\big),\ldots,F\big(x,\bar{x};\nabla f_k(\bar{x})+\nabla^2 f_k(\bar{x})p\big)$.

Remark 1. Let $\alpha(x,\bar{x})=1$. Then second order (F,α,ρ,d)-convexity becomes the second order (F,ρ)-convexity introduced by Zhang and Mond [16]. In addition, if we set second order term equal to zero, i.e., $p = 0$, it reduces to (F, ρ)-convexity [2,13].

Example. Consider the function $f: X (= R_+) \to R$ such that $f(x) = x^2 - 2x$. If, we define the functions

$$F(x, \bar{x}; a) = a(x - \bar{x}) - 4x,$$
$$d(x, \bar{x}) = x - \bar{x},$$
$$\alpha(x, \bar{x}) = \frac{x + \bar{x} + 1}{2},$$

then for $\rho = 0$, f is second order (F, α, ρ, d)-convex at $\bar{x} = 0$ with respect to p, $-\infty < p \leqq 1$.

In order to prove the strong duality theorem, we need the following Kuhn–Tucker type necessary conditions [7].

Theorem 1 (*Kuhn–Tucker type necessary conditions*). *Assume that x^* is an efficient solution for (P) at which the Kuhn–Tucker constraint qualification is satisfied. Then there exist $\lambda^* \in R^k$ and $y^* \in R^m$, such that*

$$\lambda^{*t} \nabla f(x^*) + y^{*t} \nabla g(x^*) = 0,$$
$$y^{*t} g(x^*) = 0,$$
$$y^* \geqq 0,$$
$$\lambda^* \geq 0.$$

3. Second order Mond–Weir type duality

In this section, we consider the following Mond–Weir type second order dual associated with multiobjective problem (P) and establish weak, strong and strict converse duality theorems under generalized second order (F, α, ρ, d)-convexity:

$$(\mathbf{MD}) \quad \text{Maximize} \quad f(u) - \frac{1}{2} p^t \nabla^2 f(u) p$$

$$\text{subject to} \quad \nabla \lambda^t f(u) + \nabla^2 \lambda^t f(u) p + \nabla y^t g(u) + \nabla^2 y^t g(u) p = 0, \tag{2}$$

$$y^t g(u) - \frac{1}{2} p^t \nabla^2 y^t g(u) p \geqq 0, \tag{3}$$

$$y \geqq 0, \tag{4}$$

$$\lambda \geq 0, \tag{5}$$

where λ is a k-dimensional vector, and y is an m-dimensional vector.

Theorem 2 (Weak duality). *Suppose that for all feasible x in (P) and all feasible (u, y, λ, p) in (MD)*

(i) *$y^t g(\cdot)$ is second order (F, α, ρ, d)-quasiconvex at u, and assume that any one of the following conditions holds*:

(ii) *$\lambda > 0$, and $f(\cdot)$ is strong second order (F, α_1, ρ_1, d)-pseudoconvex at u with $\alpha^{-1}\rho + \alpha_1^{-1}\rho_1\lambda \geqq 0$,*

(iii) *$\lambda^t f(\cdot)$ is strictly second order (F, α_2, ρ_2, d)-pseudoconvex at u with $\alpha^{-1}\rho + \alpha_2^{-1}\rho_2 \geqq 0$.*

Then the following cannot hold:

$$f(x) \leq f(u) - \frac{1}{2} p^t \nabla^2 f(u) p. \tag{6}$$

Proof. Let x be any feasible solution in (P) and (u, y, λ, p) be any feasible solution in (MD). Then we have

$$y^t g(x) \leqq 0 \leqq y^t g(u) - \frac{1}{2} p^t \nabla^2 y^t g(u) p. \tag{7}$$

Using second order (F, α, ρ, d)-quasiconvexity of $y^t g(\cdot)$ at u, we get

$$F\big(x, u; \alpha(x, u)\{\nabla y^t g(u) + \nabla^2 y^t g(u) p\}\big) \leqq -\rho d^2(x, u). \tag{8}$$

Since $\alpha(x, u) > 0$, the inequality (8) with the sublinearity of F yields

$$F(x, u; \nabla y^t g(u) + \nabla^2 y^t g(u) p) \leqq -\alpha^{-1}(x, u)\rho d^2(x, u). \tag{9}$$

The first dual constraint and the sublinearity of F give

$$F\big(x, u; \nabla \lambda^t f(u) + \nabla^2 \lambda^t f(u) p\big) \geqq -F\big(x, u; \nabla y^t g(u) + \nabla^2 y^t g(u) p\big). \tag{10}$$

The inequalities (9) and (10) imply

$$F\big(x, u; \nabla \lambda^t f(u) + \nabla^2 \lambda^t f(u) p\big) \geqq \alpha^{-1}(x, u)\rho d^2(x, u). \tag{11}$$

Now suppose contrary to the result that (6) holds, i.e.,

$$f(x) \leq f(u) - \frac{1}{2} p^t \nabla^2 f(u) p, \tag{12}$$

which by virtue of (ii), leads to

$$F\big(x, u; \alpha_1(x, u)\{\nabla f(u) + \nabla^2 f(u) p\}\big) \leq -\rho_1 d^2(x, u). \tag{13}$$

On multiplying (13) by $\lambda > 0$ and using the sublinearity of F with $\alpha_1(x, u) > 0$, we obtain

$$F\big(x, u; \nabla \lambda^t f(u) + \nabla^2 \lambda^t f(u) p\big) < -\alpha_1^{-1}(x, u)\rho_1 \lambda d^2(x, u) \leqq \alpha^{-1}(x, u)\rho d^2(x, u),$$

which contradicts (11). Hence (6) cannot hold.

On the other hand, multiplying the inequality (12) by λ, we have

$$\lambda^t f(x) \leqq \lambda^t f(u) - \frac{1}{2} p^t \nabla^2 \lambda^t f(u) p. \tag{14}$$

When hypothesis (iii) holds, the inequality (14) implies

$$F\big(x, u; \alpha_2(x, u)\{\nabla \lambda^t f(u) + \nabla^2 \lambda^t f(u) p\}\big) < -\rho_2 d^2(x, u). \tag{15}$$

Since F is sublinear and $\alpha_2(x, u) > 0$, it follows from (15) that

$$F\big(x, u; \nabla \lambda^t f(u) + \nabla^2 \lambda^t f(u) p\big) < -\alpha_2^{-1}(x, u)\rho_2 d^2(x, u) \leqq \alpha^{-1}(x, u)\rho d^2(x, u),$$

a contradiction to (11). Hence (6) cannot hold. □

Theorem 3 (Strong duality). *Let $\bar{x}$ be an efficient solution of (P) at which the Kuhn–Tucker constraint qualification is satisfied. Then there exist $\bar{y} \in R^m$ and $\bar{\lambda} \in R^k$, such that $(\bar{x}, \bar{y}, \bar{\lambda}, \bar{p} = 0)$ is feasible for (MD) and the corresponding values of (P) and (MD) are equal.*

If, in addition, the assumptions of weak duality (Theorem 2) hold for all feasible solutions of (P) and (MD), then $(\bar{x}, \bar{y}, \bar{\lambda}, \bar{p} = 0)$ is an efficient solution of (MD).

Proof. Since $\bar{x}$ is an efficient solution of (P) at which the Kuhn–Tucker constraint qualification is satisfied, then by Theorem 1, there exist $\bar{y} \in R^m$ and $\bar{\lambda} \in R^k$, such that

$$\bar{\lambda}^t \nabla f(\bar{x}) + \bar{y}^t \nabla g(\bar{x}) = 0,$$
$$\bar{y}^t g(\bar{x}) = 0,$$
$$\bar{y} \geqq 0,$$
$$\bar{\lambda} \geq 0.$$

Therefore $(\bar{x}, \bar{y}, \bar{\lambda}, \bar{p} = 0)$ is feasible for (MD) and the corresponding values of (P) and (MD) are equal. The efficiency of this feasible solution for (MD) thus follows from weak duality (Theorem 2). □

Theorem 4 (Strict converse duality). *Let $\bar{x}$ and $(\bar{u}, \bar{y}, \bar{\lambda}, \bar{p})$ be the efficient solutions of (P) and (MD), respectively, such that*

$$\bar{\lambda}^t f(\bar{x}) = \bar{\lambda}^t f(\bar{u}) - \tfrac{1}{2} \bar{p}^t \nabla^2 \bar{\lambda}^t f(\bar{u}) \bar{p}. \tag{16}$$

Suppose that any one of the following conditions is satisfied:

(i) *$\bar{y}^t g(\cdot)$ is second order (F, α, ρ, d)-quasiconvex at $\bar{u}$ and $\bar{\lambda}^t f(\cdot)$ is strictly second order (F, α_1, ρ_1, d)-pseudoconvex at $\bar{u}$ with $\alpha^{-1}\rho + \alpha_1^{-1}\rho_1 \geqq 0$,*

(ii) $\bar{y}^t g(\cdot)$ *is strictly second order* (F, α, ρ, d)*-pseudoconvex at* $\bar{u}$ *and* $\bar{\lambda}^t f(\cdot)$ *is second order* (F, α_1, ρ_1, d)*-quasiconvex at* $\bar{u}$ *with* $\alpha^{-1}\rho + \alpha_1^{-1}\rho_1 \geqq 0$.

Then $\bar{x} = \bar{u}$; *that is,* $\bar{u}$ *is an efficient solution of* (P).

Proof. We assume that $\bar{x} \neq \bar{u}$ and reach a contradiction. Since $\bar{x}$ and $(\bar{u}, \bar{y}, \bar{\lambda}, \bar{p})$ are, respectively, the feasible solutions of (P) and (MD), we have

$$\bar{y}^t g(\bar{x}) \leqq 0 \leqq \bar{y}^t g(\bar{u}) - \frac{1}{2}\bar{p}^t \nabla^2 \bar{y}^t g(\bar{u})\bar{p}. \tag{17}$$

Using second order (F, α, ρ, d)-quasiconvexity of $\bar{y}^t g(\cdot)$ at $\bar{u}$, we get

$$F\left(\bar{x}, \bar{u}; \alpha(\bar{x}, \bar{u})\left\{\nabla \bar{y}^t g(\bar{u}) + \nabla^2 \bar{y}^t g(\bar{u})\bar{p}\right\}\right) \leqq -\rho d^2(\bar{x}, \bar{u}). \tag{18}$$

Since $\alpha(\bar{x}, \bar{u}) > 0$, the inequality (18) along with the sublinearity of F yields

$$F\left(\bar{x}, \bar{u}; \nabla \bar{y}^t g(\bar{u}) + \nabla^2 \bar{y}^t g(\bar{u})\bar{p}\right) \leqq -\alpha^{-1}(\bar{x}, \bar{u})\rho d^2(\bar{x}, \bar{u}). \tag{19}$$

The first dual constraint and the sublinearity of F imply

$$\begin{aligned} &F\left(\bar{x}, \bar{u}; \nabla \bar{\lambda}^t f(\bar{u}) + \nabla^2 \bar{\lambda}^t f(\bar{u})\bar{p}\right) + F\left(\bar{x}, \bar{u}; \nabla \bar{y}^t g(\bar{u}) + \nabla^2 \bar{y}^t g(\bar{u})\bar{p}\right) \\ &\quad \geqq F\left(\bar{x}, \bar{u}; \nabla \bar{\lambda}^t f(\bar{u}) + \nabla^2 \bar{\lambda}^t f(\bar{u})\bar{p} + \nabla \bar{y}^t g(\bar{u}) + \nabla^2 \bar{y}^t g(\bar{u})\bar{p}\right) = 0. \end{aligned} \tag{20}$$

The inequalities (19), (20) and $\alpha^{-1}\rho + \alpha_1^{-1}\rho_1 \geqq 0$ imply

$$F\left(\bar{x}, \bar{u}; \nabla \bar{\lambda}^t f(\bar{u}) + \nabla^2 \bar{\lambda}^t f(\bar{u})\bar{p}\right) \geqq -\alpha_1^{-1}(\bar{x}, \bar{u})\rho_1 d^2(\bar{x}, \bar{u}). \tag{21}$$

Using strict second order (F, α_1, ρ_1, d)-pseudoconvexity of $\bar{\lambda}^t f(\cdot)$ with $\alpha(\bar{x}, \bar{u}) > 0$

$$\bar{\lambda}^t f(\bar{x}) > \bar{\lambda}^t f(\bar{u}) - \frac{1}{2}\bar{p}^t \nabla^2 \bar{\lambda}^t f(\bar{u})\bar{p},$$

contradicting (16).

When the hypothesis (ii) holds, it follows from (17) that

$$F\left(\bar{x}, \bar{u}; \alpha(\bar{x}, \bar{u})\left\{\nabla \bar{y}^t g(\bar{u}) + \nabla^2 \bar{y}^t g(\bar{u})\bar{p}\right\}\right) < -\rho d^2(\bar{x}, \bar{u}).$$

Since $\alpha(\bar{x}, \bar{u}) > 0$, the above inequality with the sublinearity of F gives

$$F\left(\bar{x}, \bar{u}; \nabla \bar{y}^t g(\bar{u}) + \nabla^2 \bar{y}^t g(\bar{u})\bar{p}\right) < -\alpha^{-1}(\bar{x}, \bar{u})\rho d^2(\bar{x}, \bar{u}),$$

which on using first dual constraint with the sublinearity of F implies

$$F\left(\bar{x}, \bar{u}; \nabla \bar{\lambda}^t f(\bar{u}) + \nabla^2 \bar{\lambda}^t f(\bar{u})\bar{p}\right) > \alpha^{-1}(\bar{x}, \bar{u})\rho d^2(\bar{x}, \bar{u}).$$

As $\alpha^{-1}\rho + \alpha_1^{-1}\rho_1 \geqq 0$, we obtain

$$F\left(\bar{x}, \bar{u}; \nabla \bar{\lambda}^t f(\bar{u}) + \nabla^2 \bar{\lambda}^t f(\bar{u})\bar{p}\right) > -\alpha_1^{-1}(\bar{x}, \bar{u})\rho_1 d^2(\bar{x}, \bar{u}). \tag{22}$$

The second order (F, α_1, ρ_1, d)-quasiconvexity of $\bar{\lambda}^t f(\cdot)$ and (22) with $\alpha_1(\bar{x}, \bar{u}) > 0$ yield

$$\bar{\lambda}^t f(\bar{x}) > \bar{\lambda}^t f(\bar{u}) - \frac{1}{2}\bar{p}^t \nabla^2 \bar{\lambda}^t f(\bar{u})\bar{p},$$

again contradicting (16). □

Remark 2. It may be noted that the strong and strict converse duality theorems are valid for weak efficient solutions as well. In the proof of strict converse duality theorem, the efficiency of $\bar{x}$ for (P) has not been used while the strong duality theorem holds for weak efficient solutions on using Theorem B in [12] instead of Theorem 1.

4. Conclusion

In this paper, second order (F, α, ρ, d)-convexity and its generalizations are introduced, which include many other generalized convexity concepts in mathematical programming as special cases. Our concepts are suitable to discuss the weak, strong and strict converse duality theorems for Mond–Weir type second order dual of multiobjective programming problem. These results can be further generalized to a class of nondifferentiable multiobjective programming problems.

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